Lecture 11

Disjoint-Set Data Structure

Source: Introduction to Algorithms, CLRS

Finding Friend Groups on Facebook

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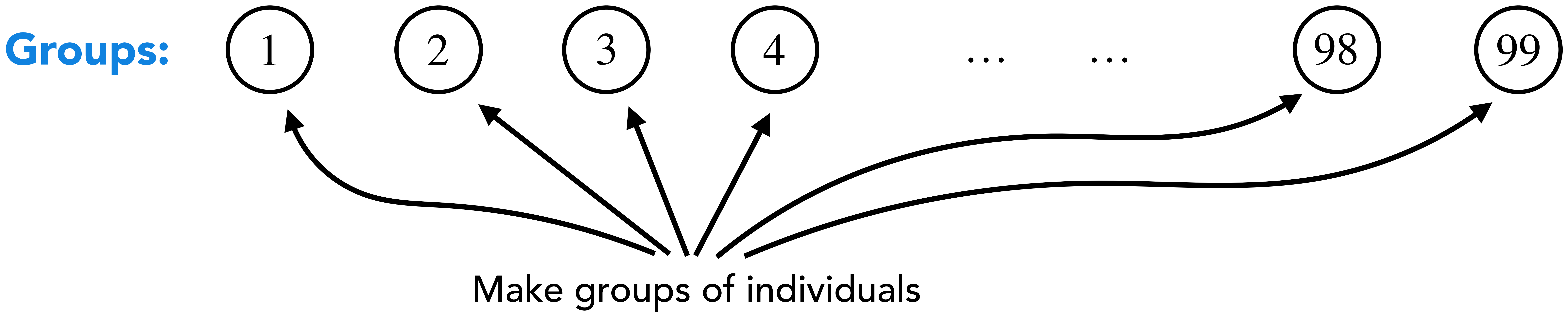
Users:

Finding Friend Groups on Facebook

Users: 1 2 3 4 … … 98 99

Finding Friend Groups on Facebook



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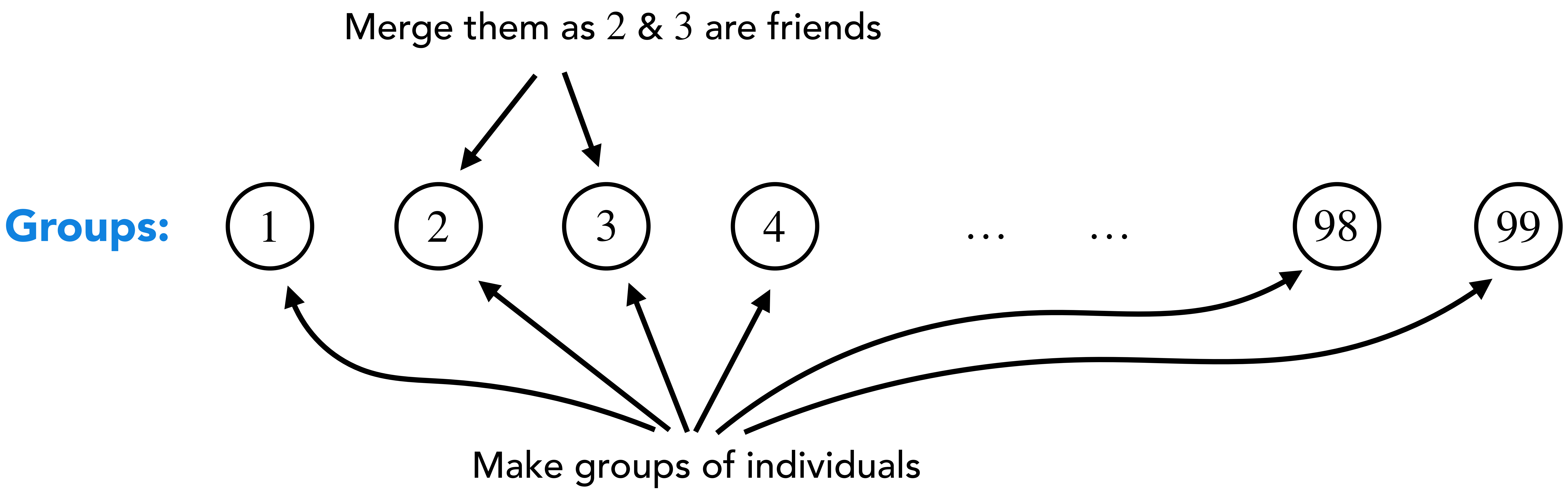

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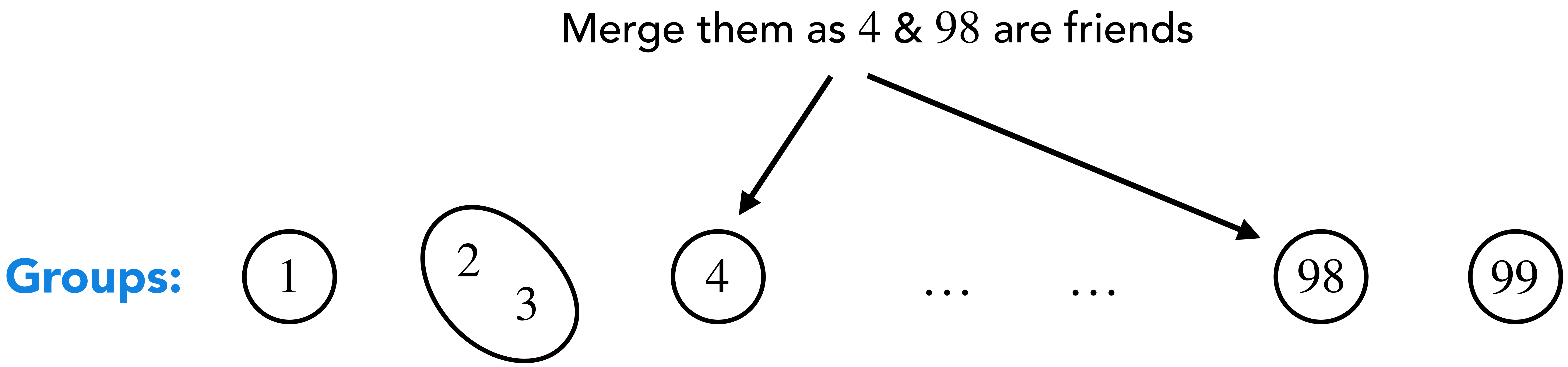

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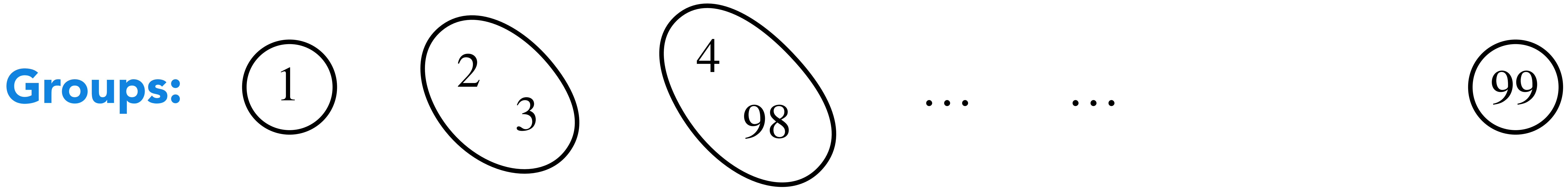

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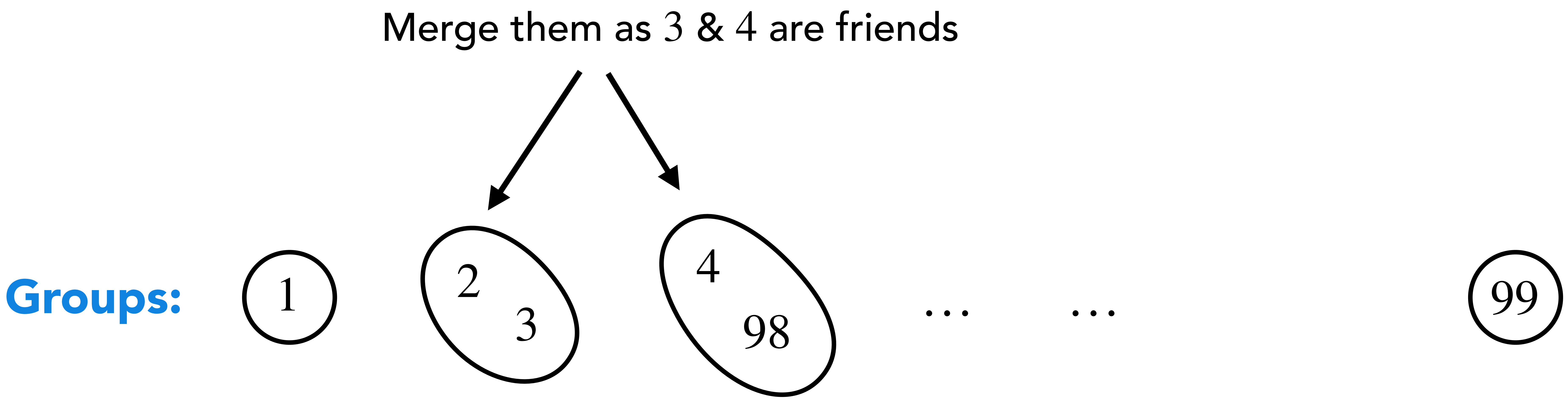

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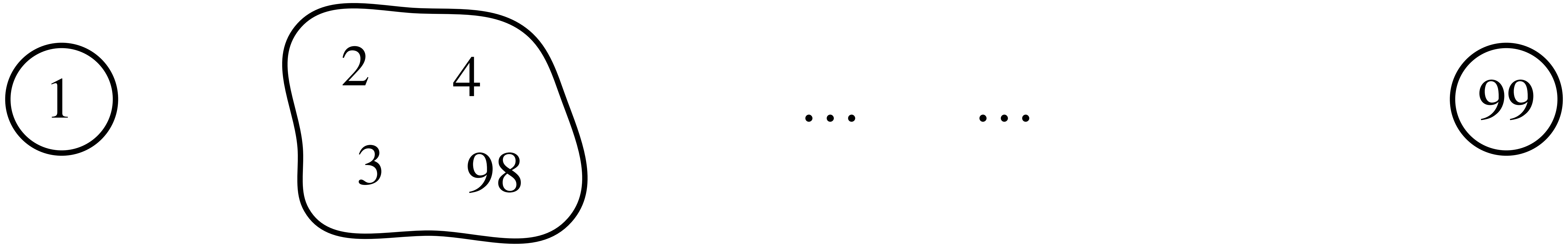

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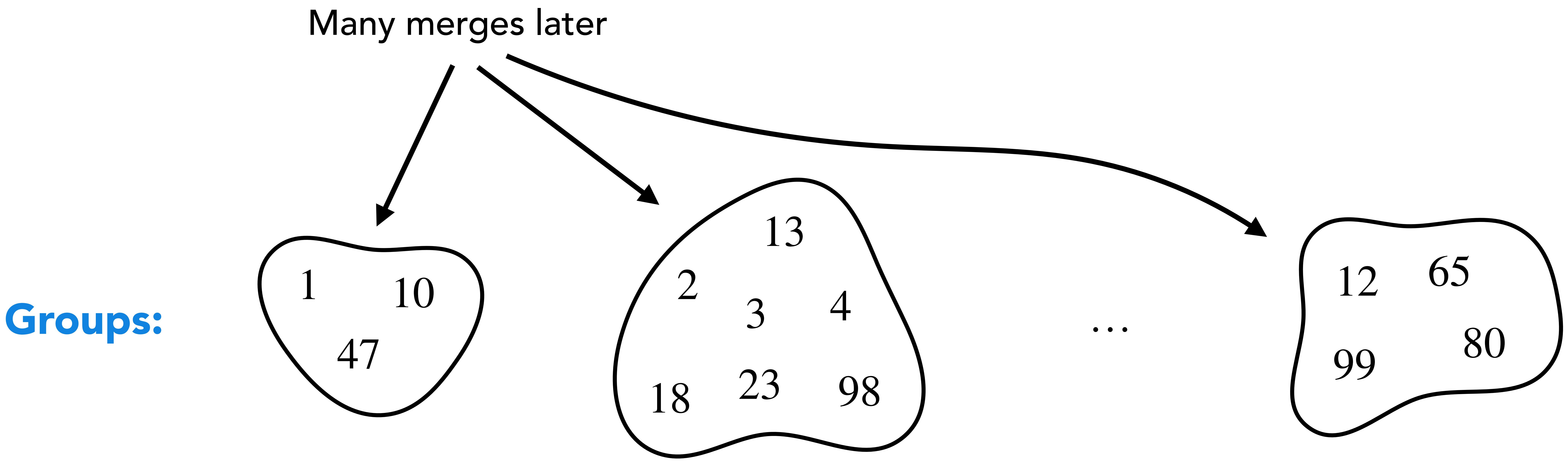

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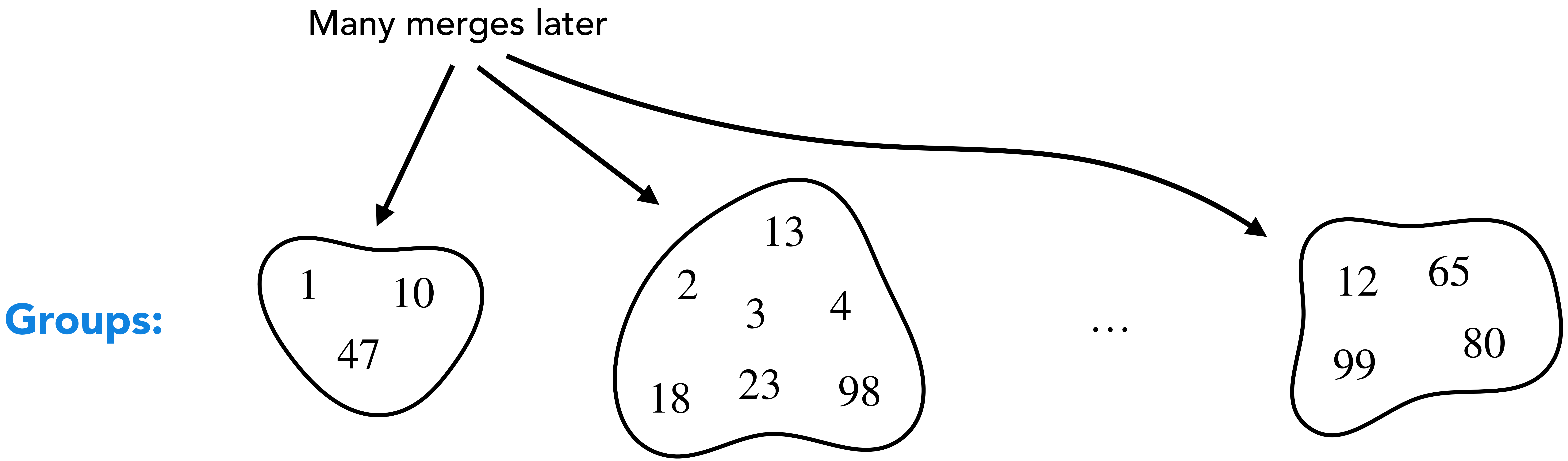

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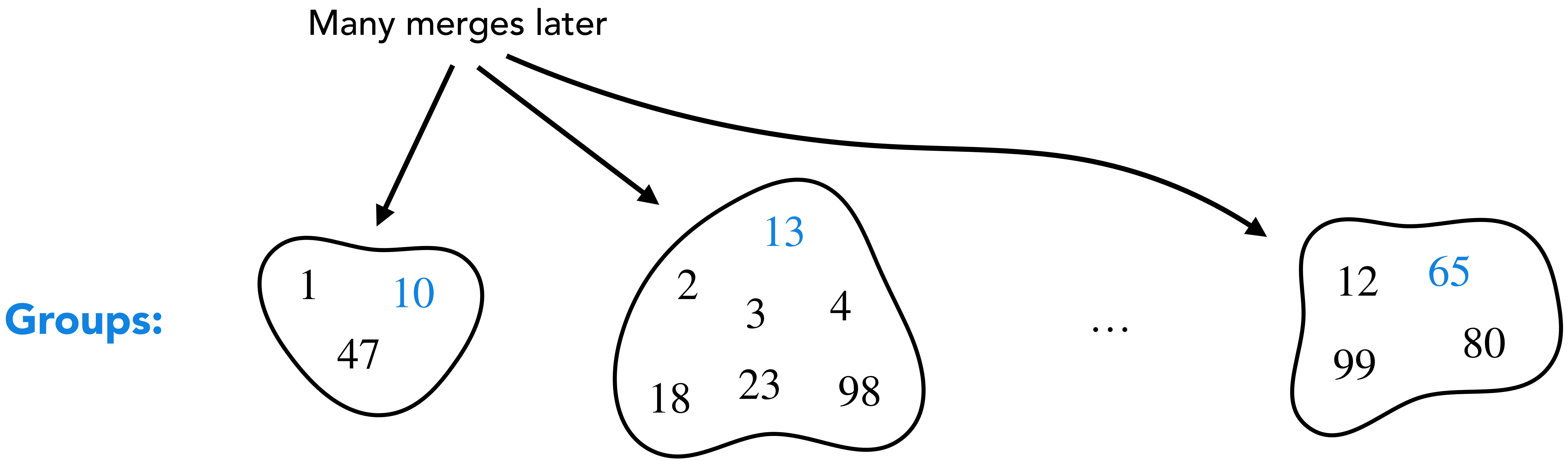

There should be a way to tell to which group a user belongs.

We achieve that by having a representative for every group.

Finding Friend Groups on Facebook

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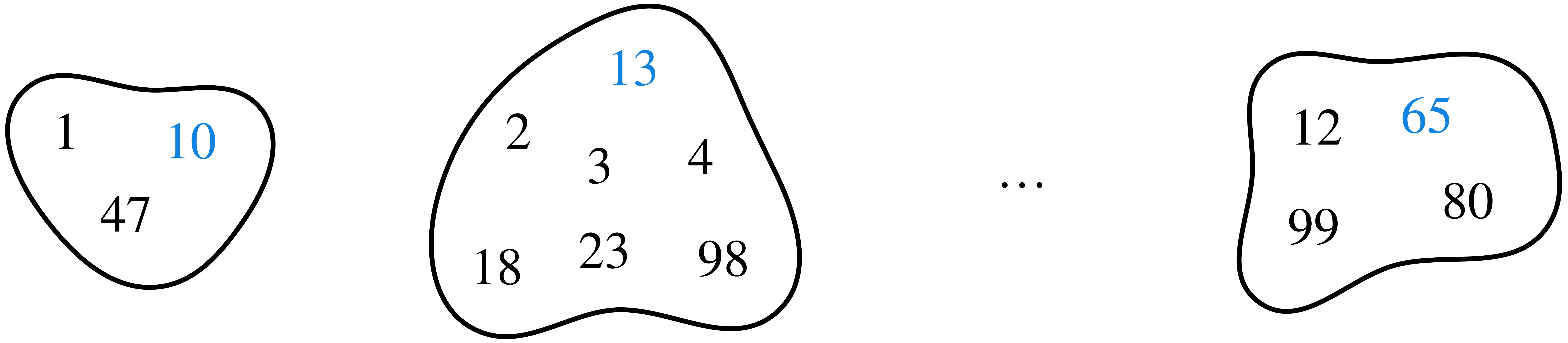

Goal: Design a data-structure so that:

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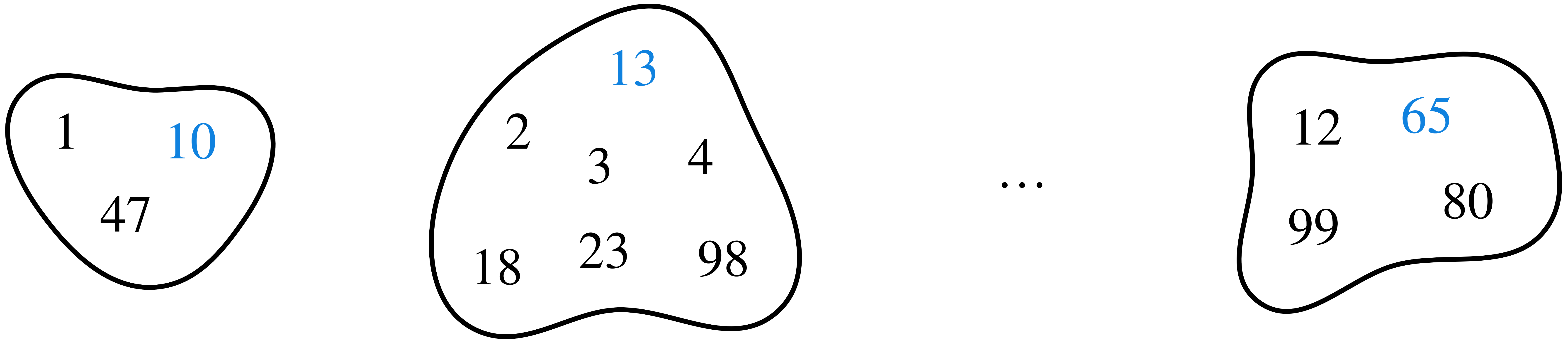

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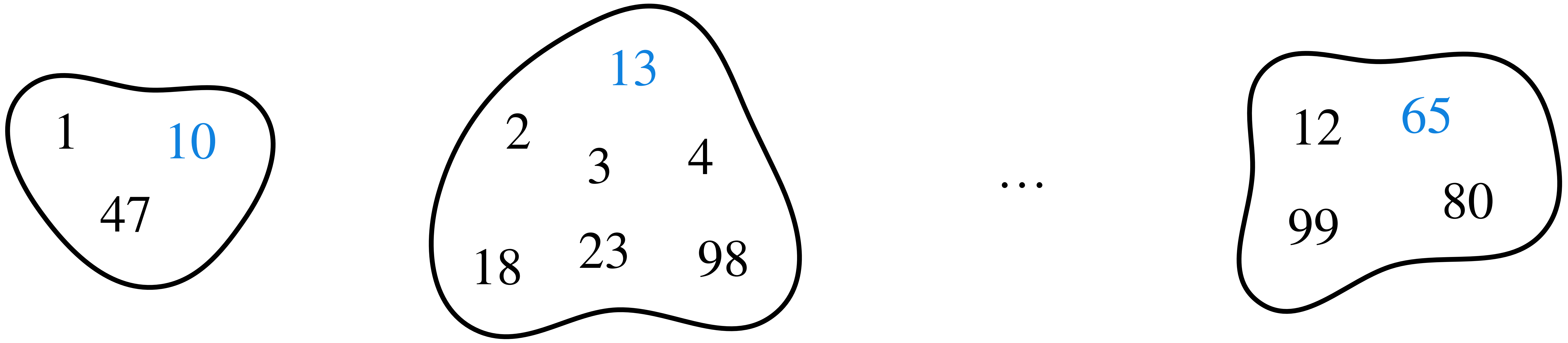

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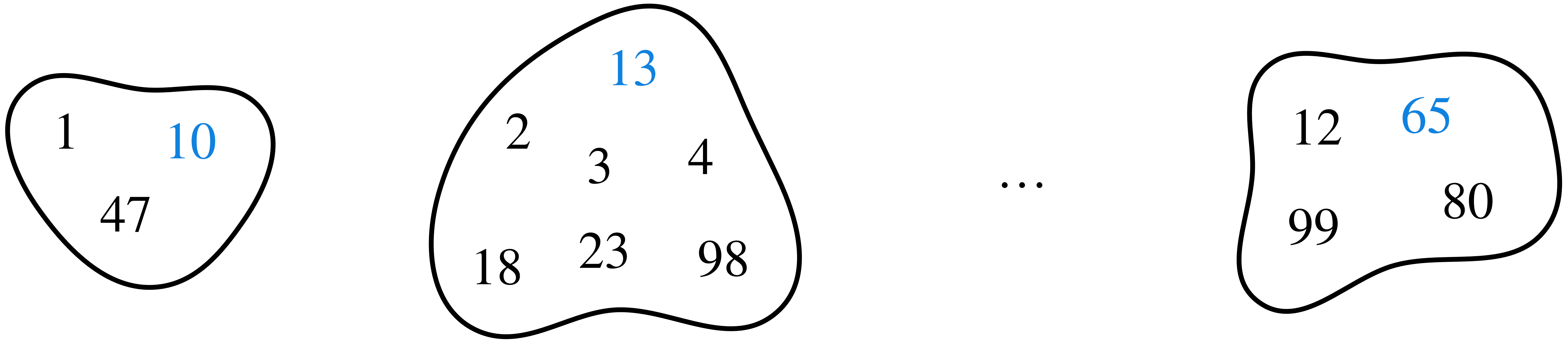

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Instead of optimising the cost of individual operations we will optimise a sequence of such operations.

Disjoint-set Data Structure

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- **Find-Set**(x): Gives the representative of the unique set that contains x.

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- Kruskal's algorithms to find minimum spanning tree.
- Finding systems on the same network, etc.

Disjoint-Sets as Linked Lists

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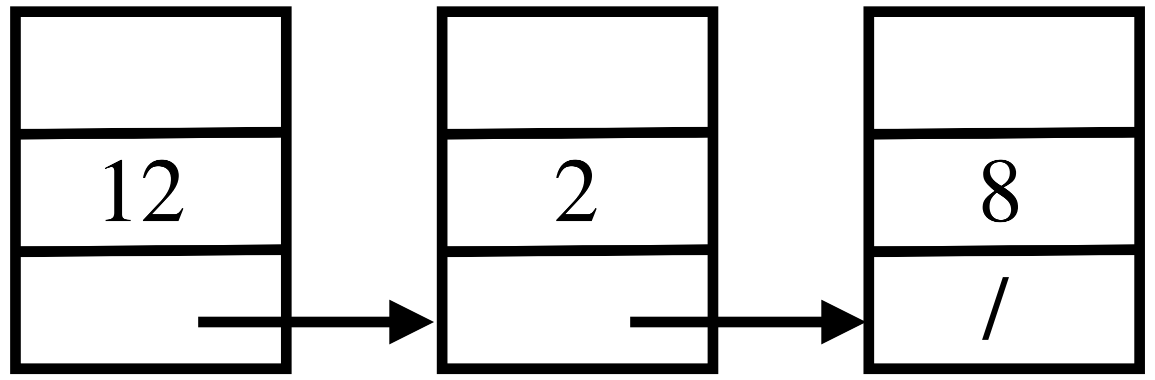
S_1



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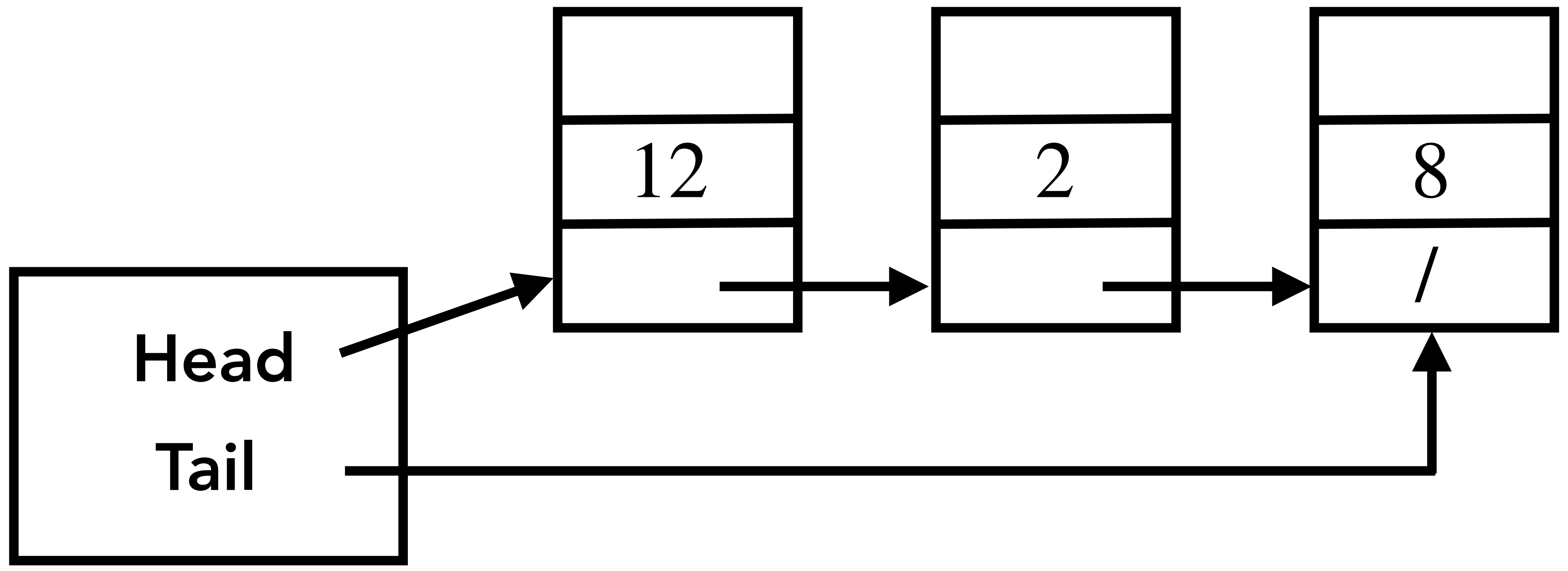

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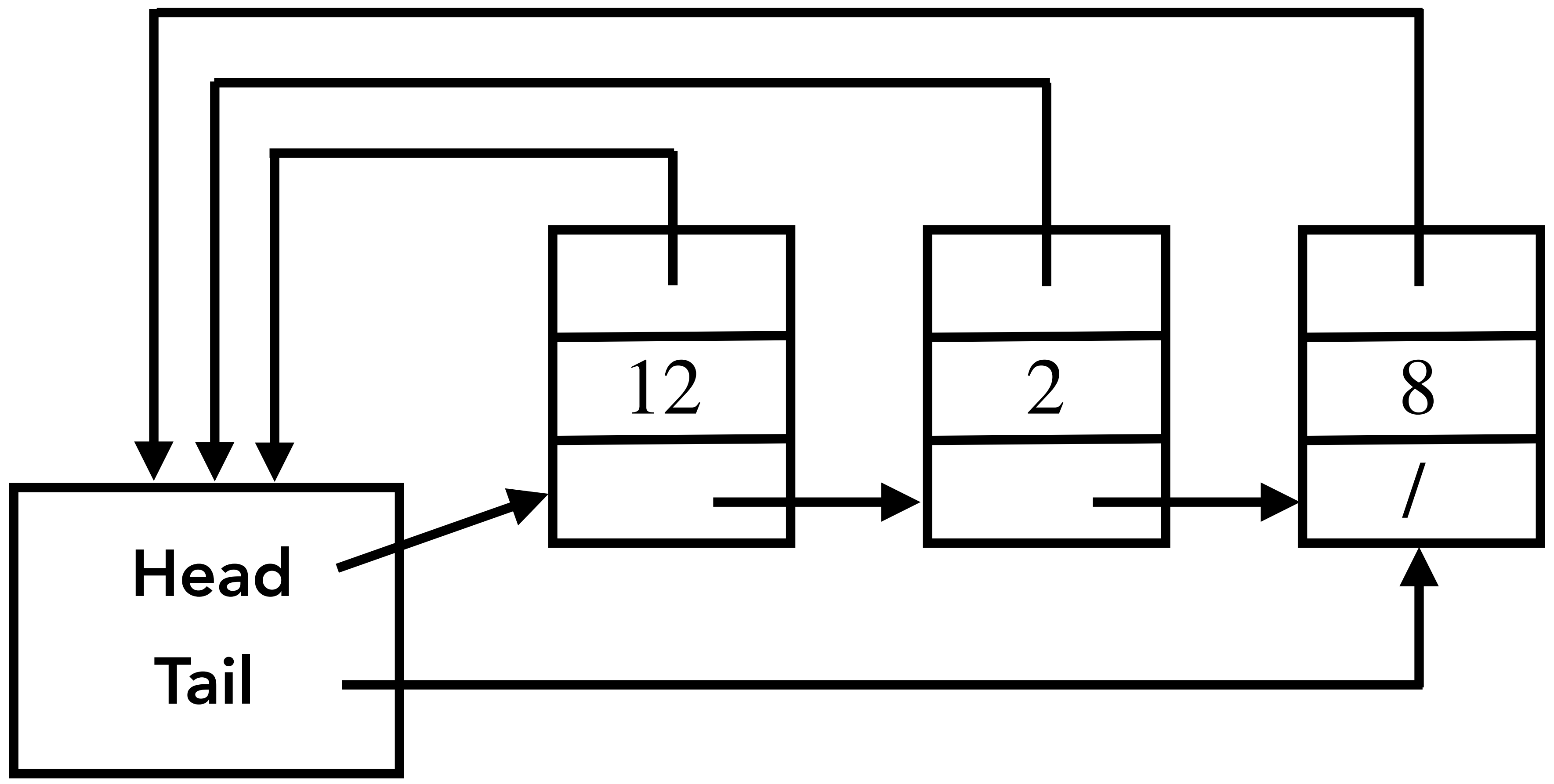

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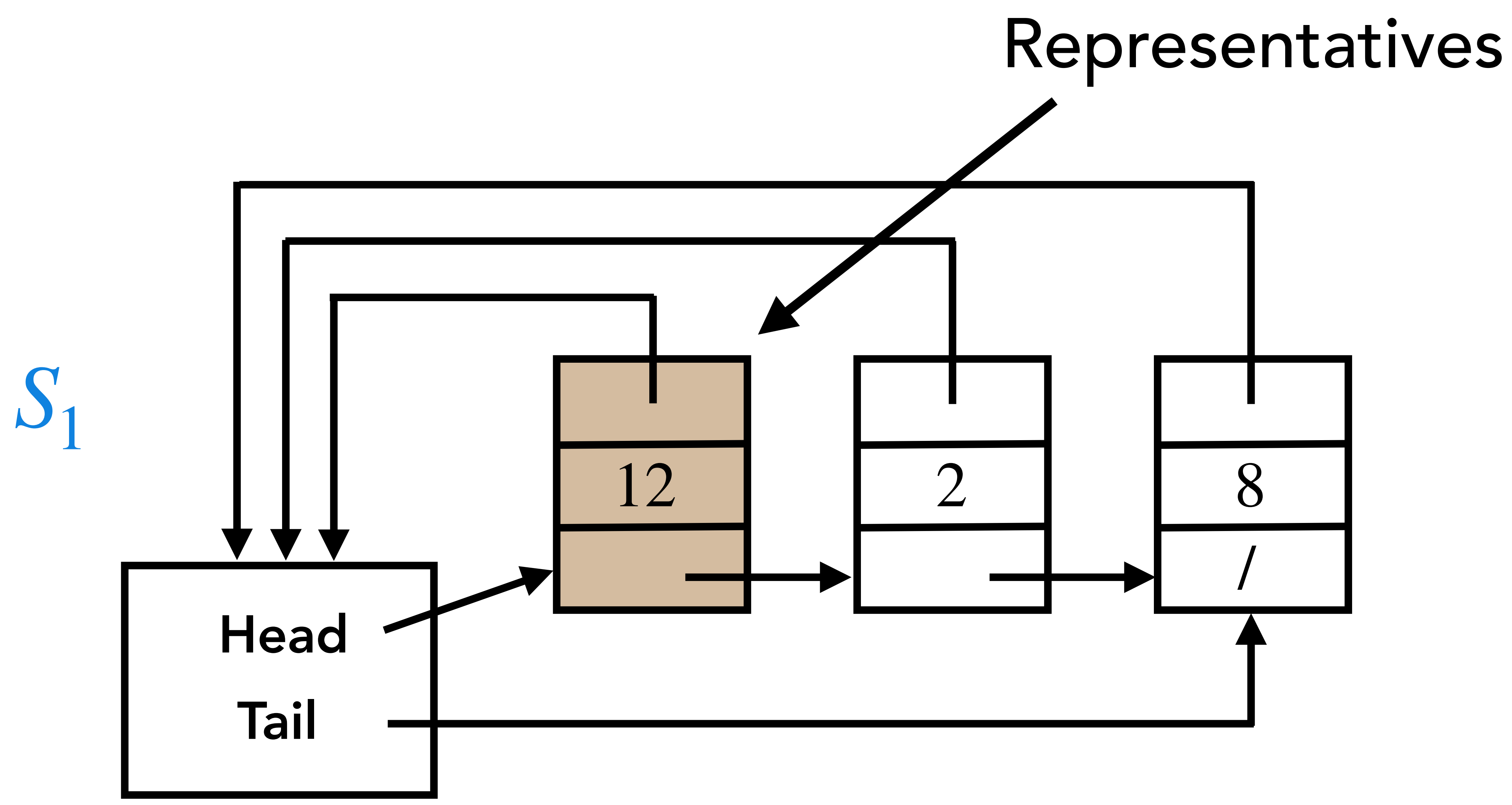

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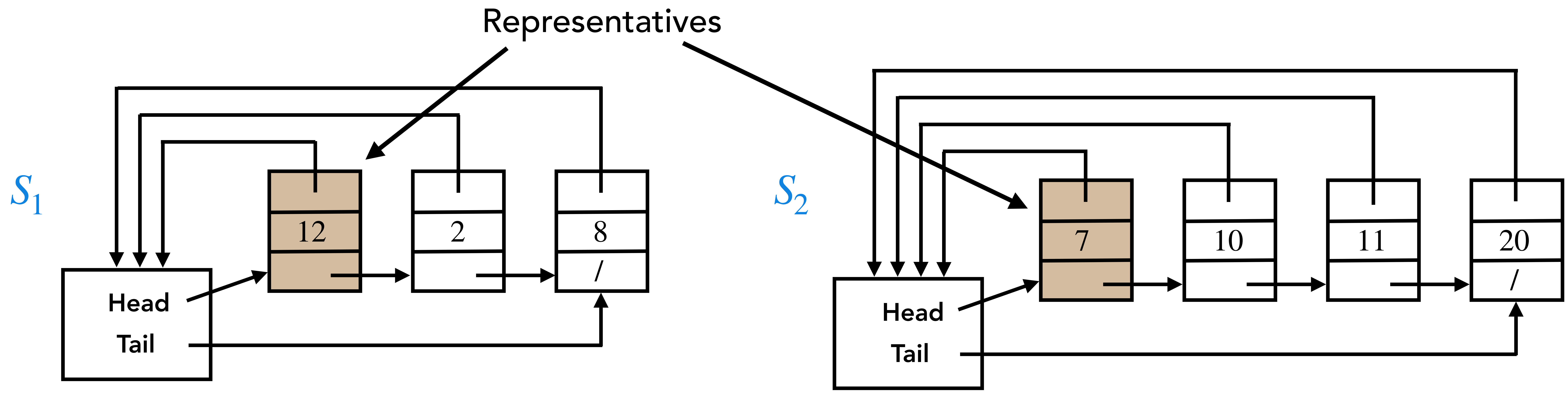

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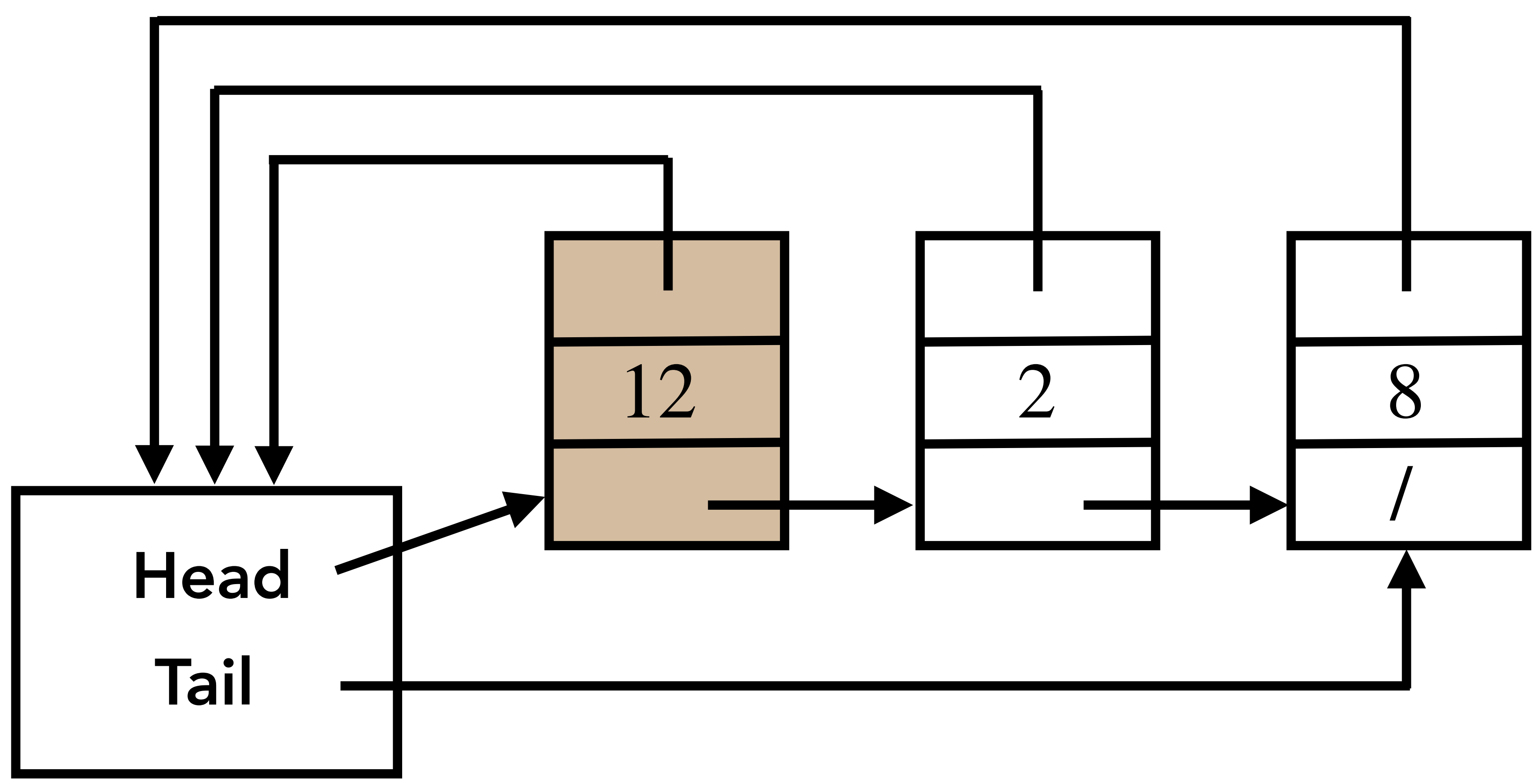
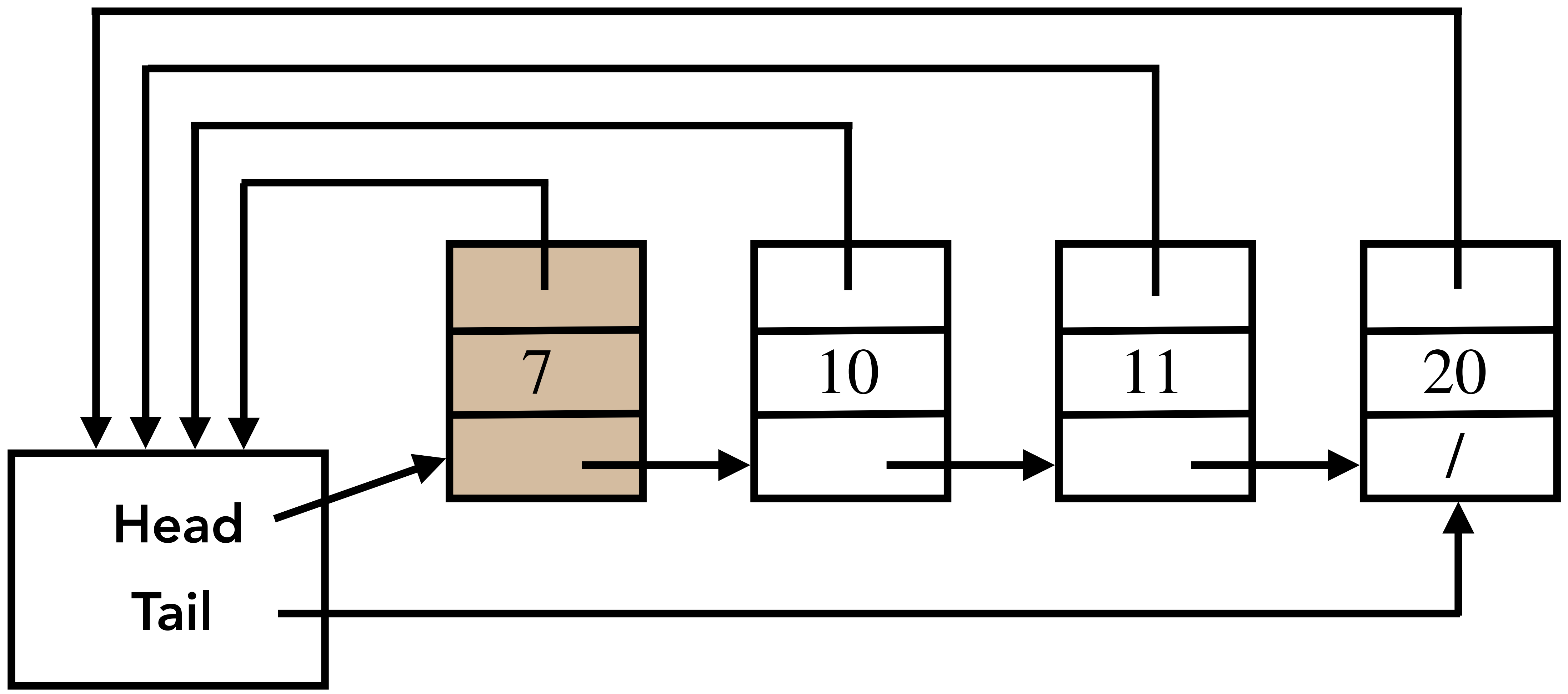


S_2



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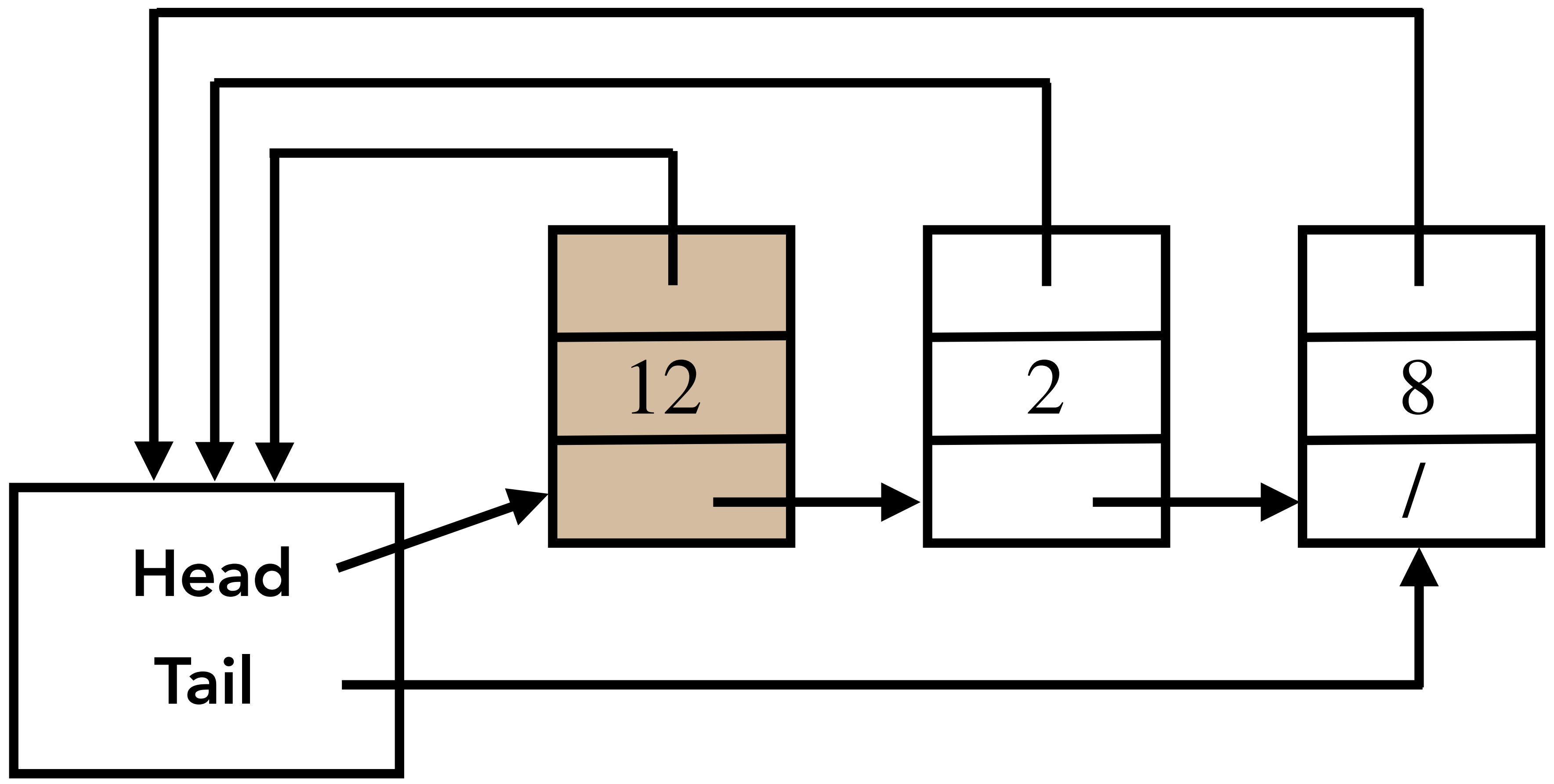

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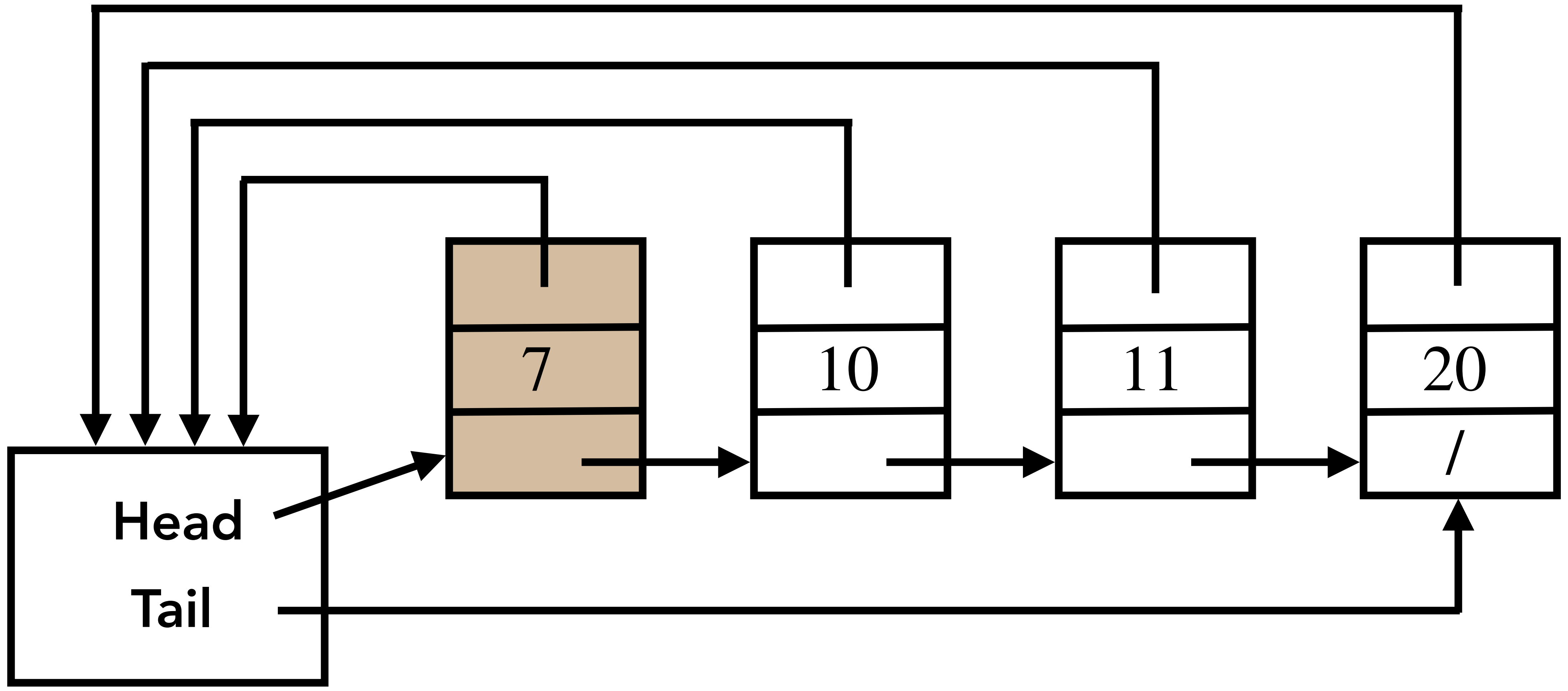
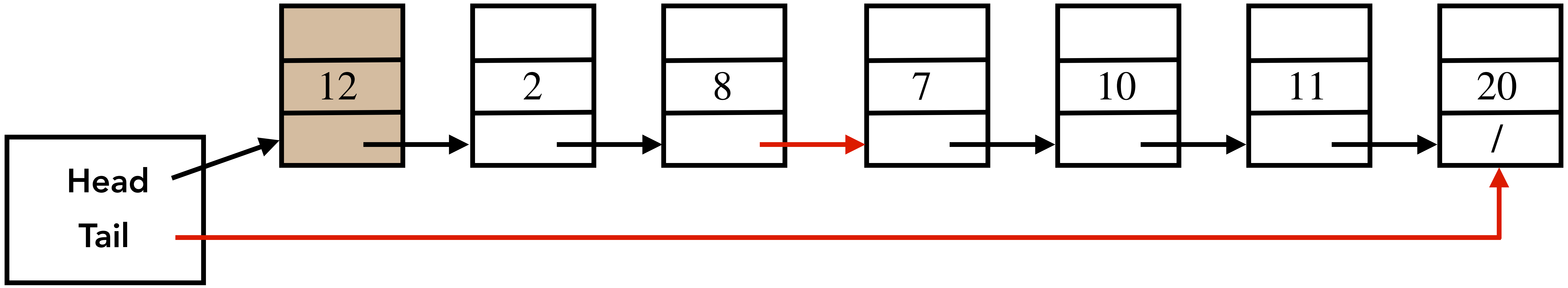


$S_1 \cup S_2$



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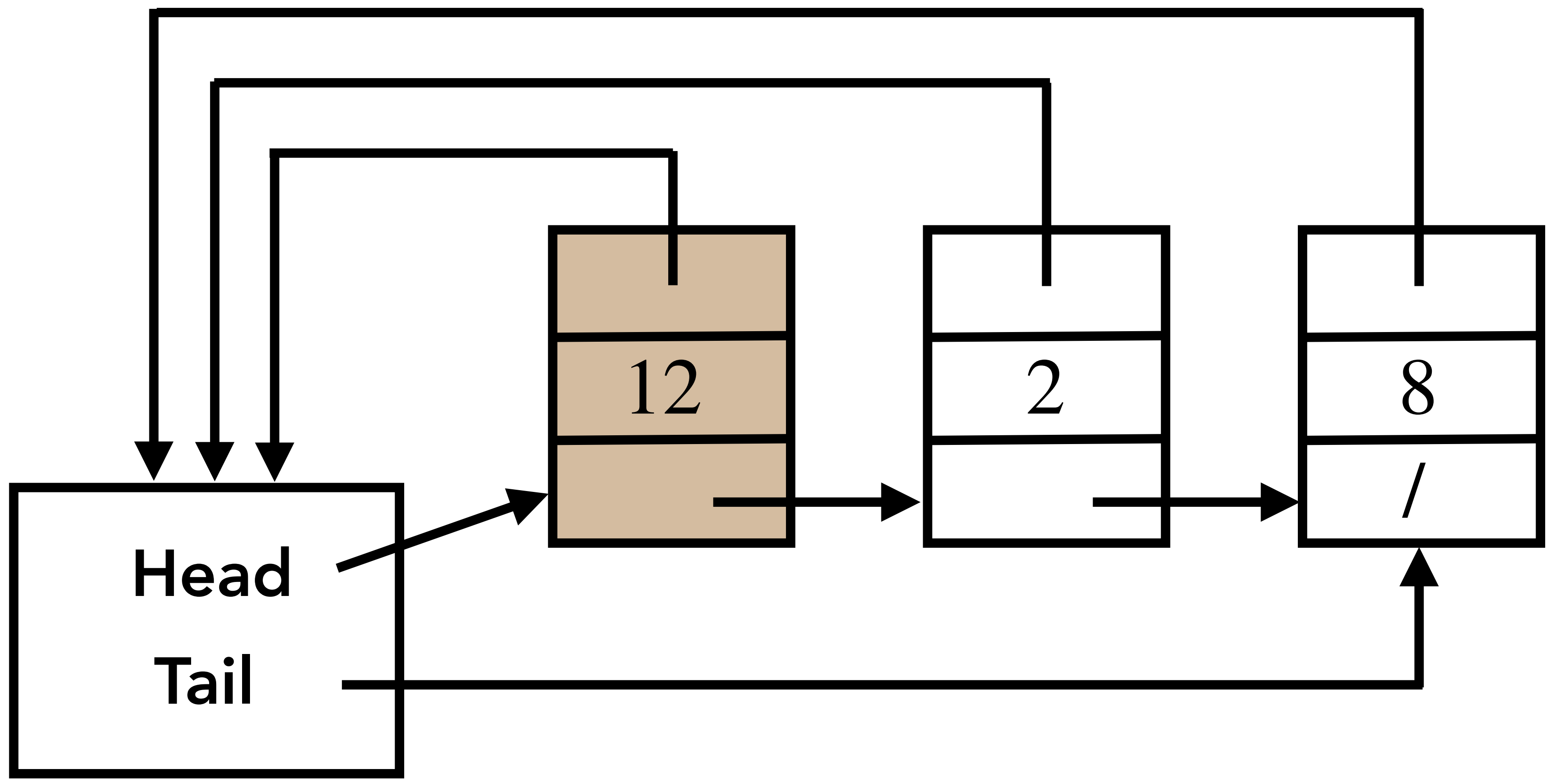

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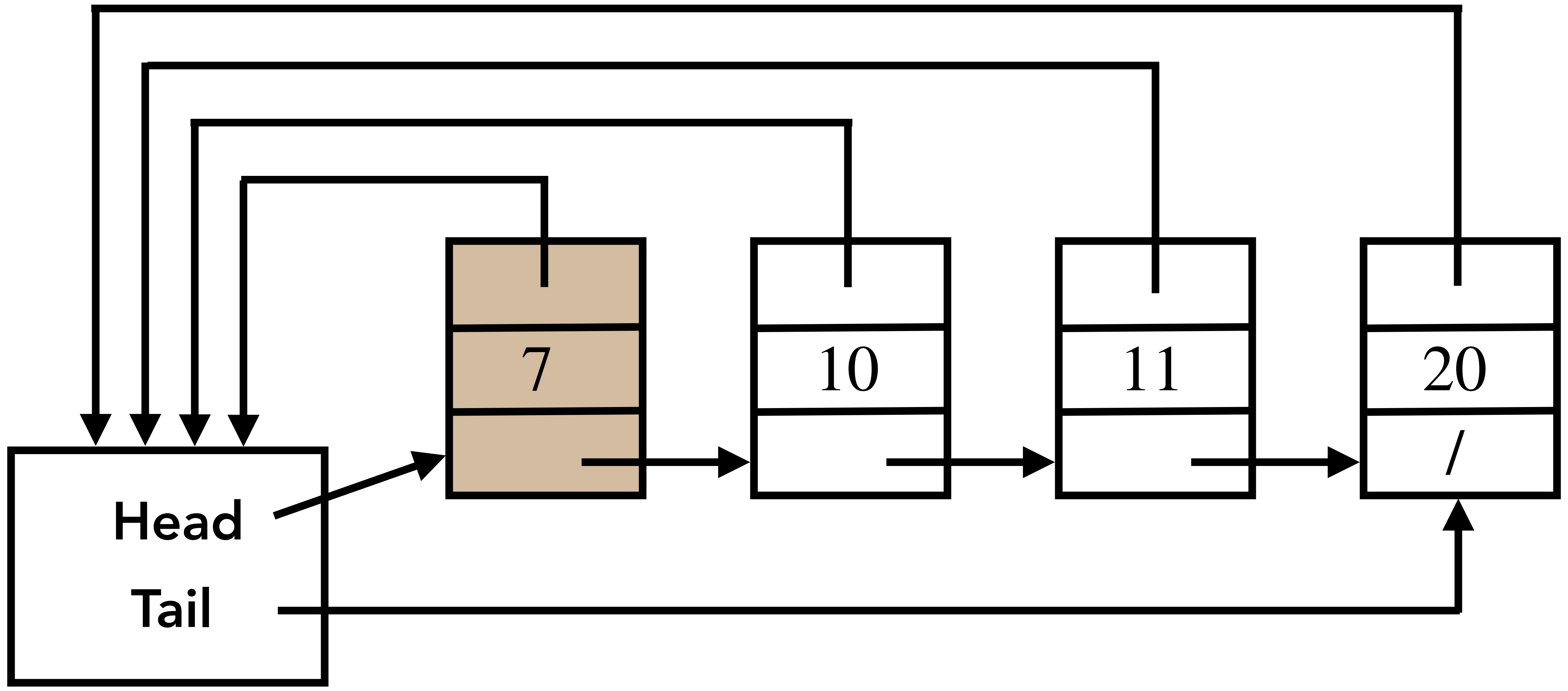

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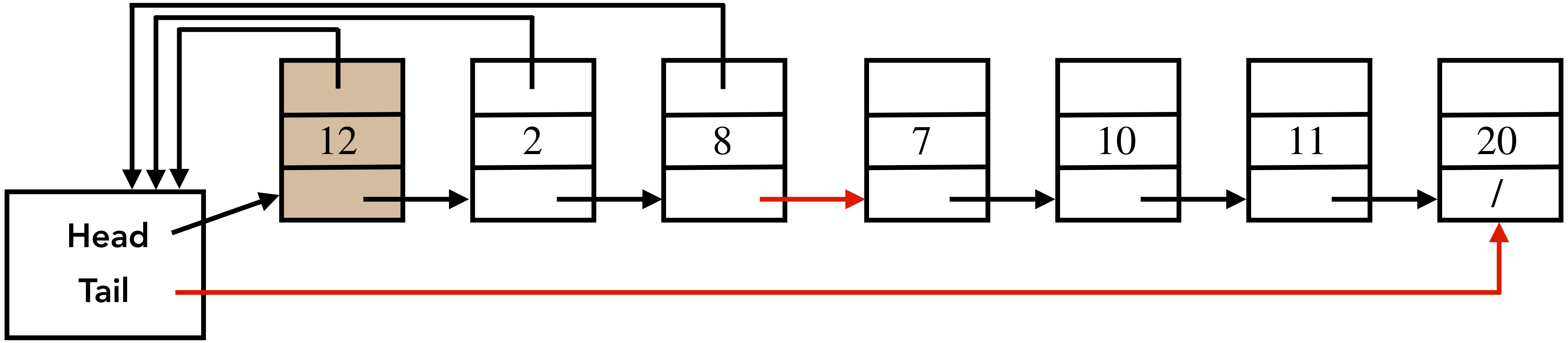

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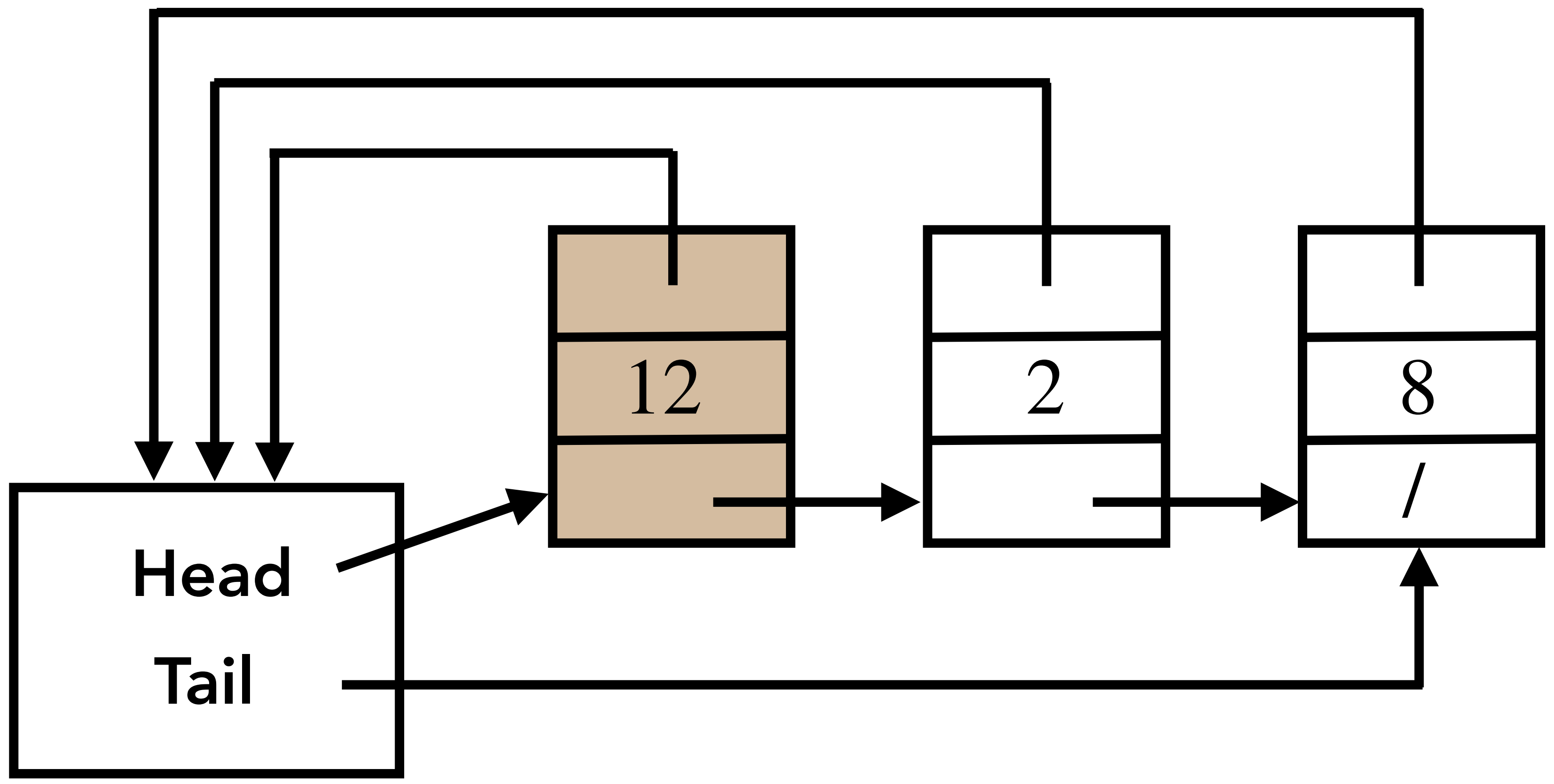

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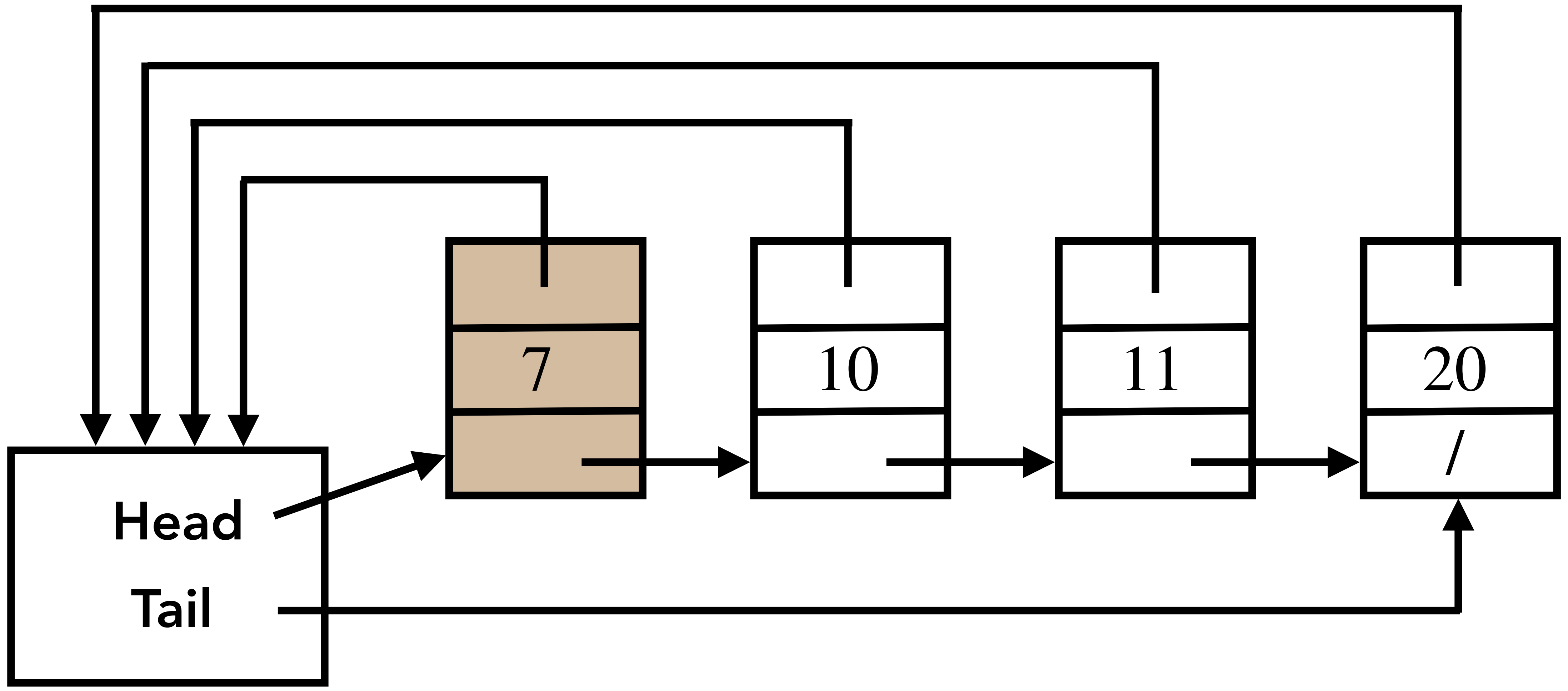

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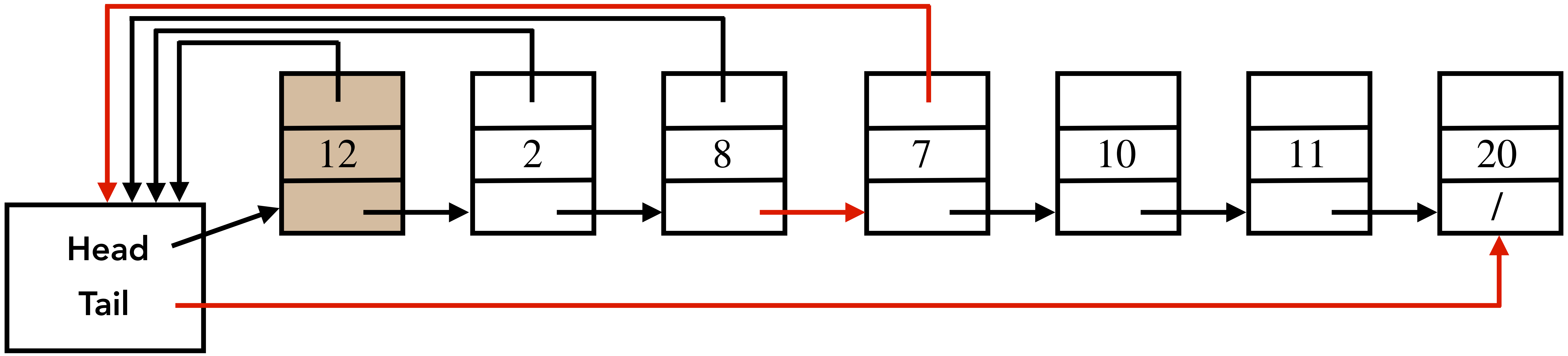

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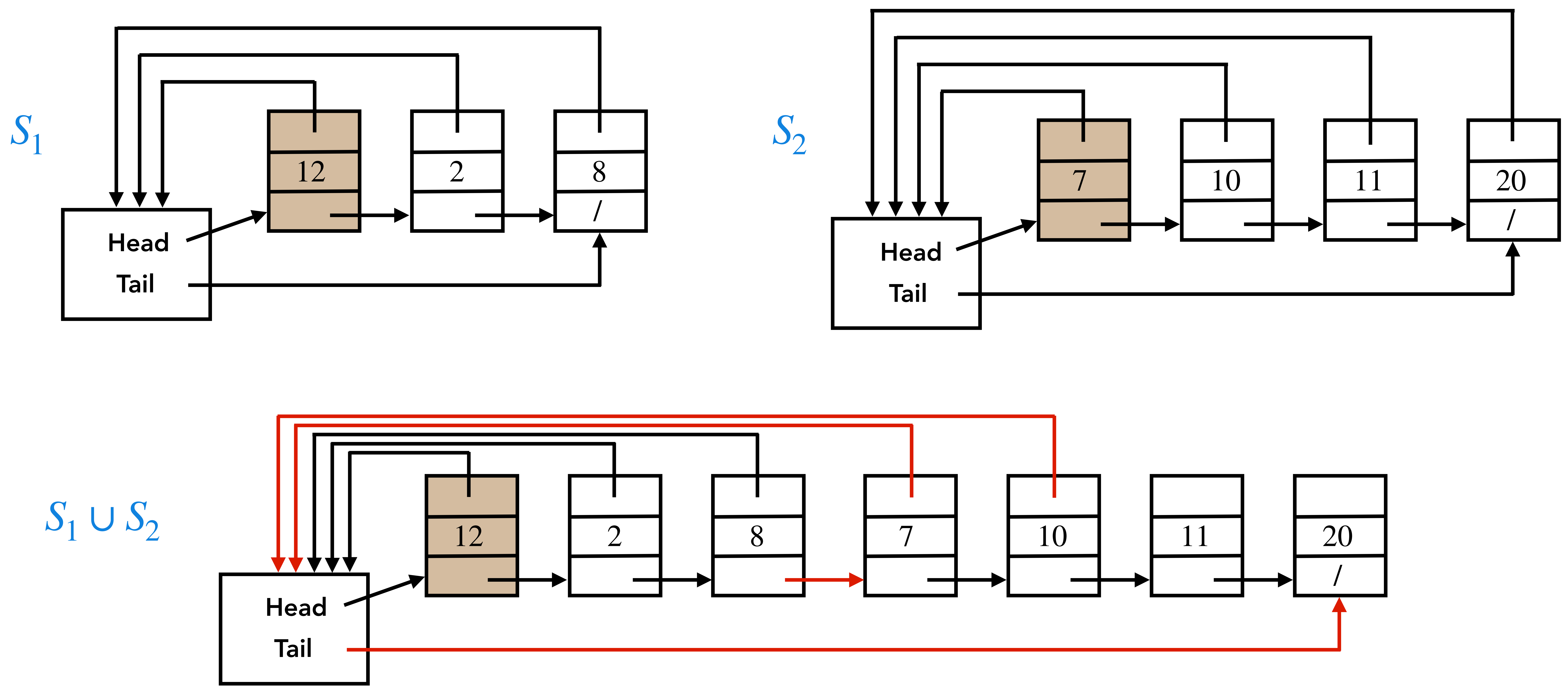

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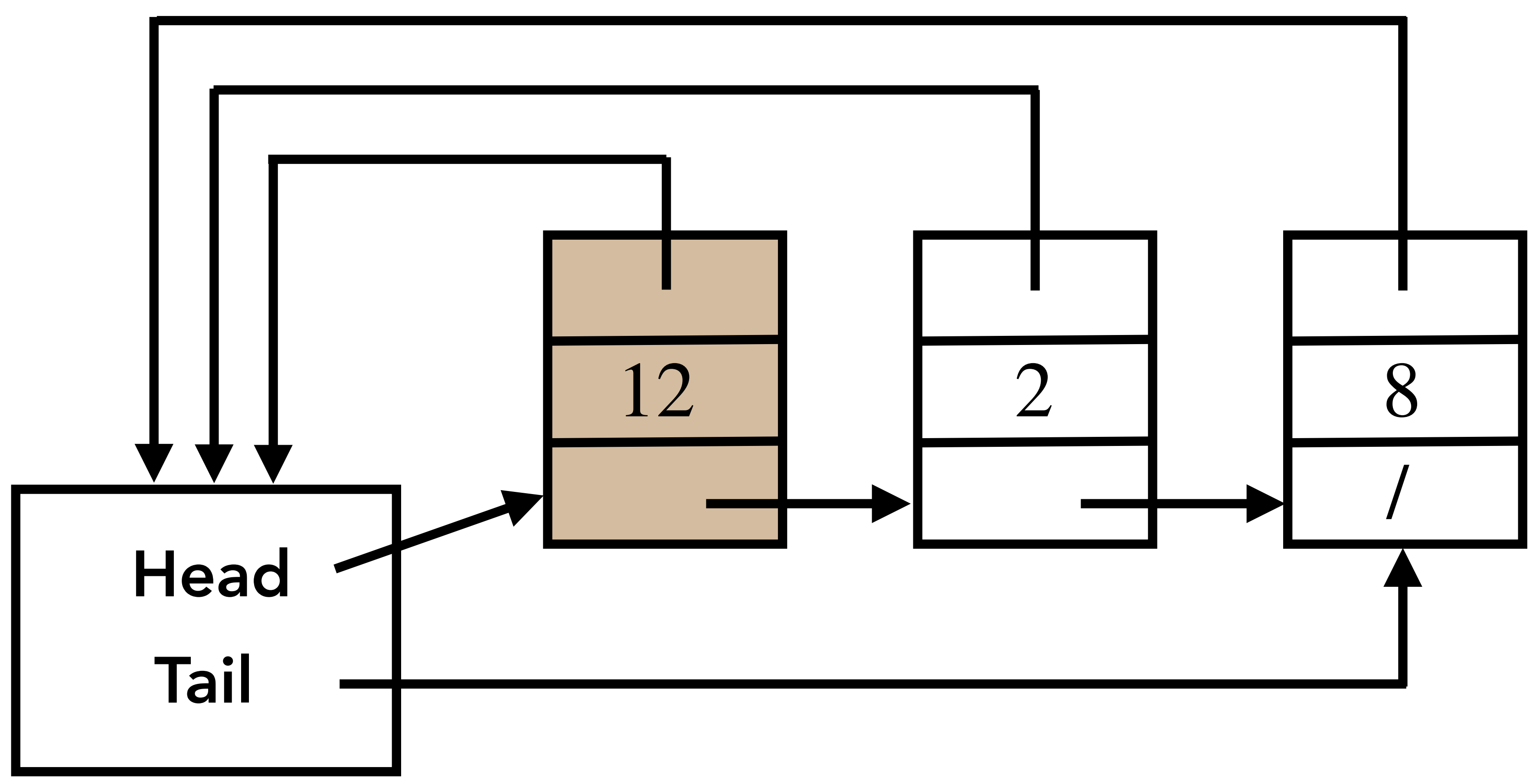

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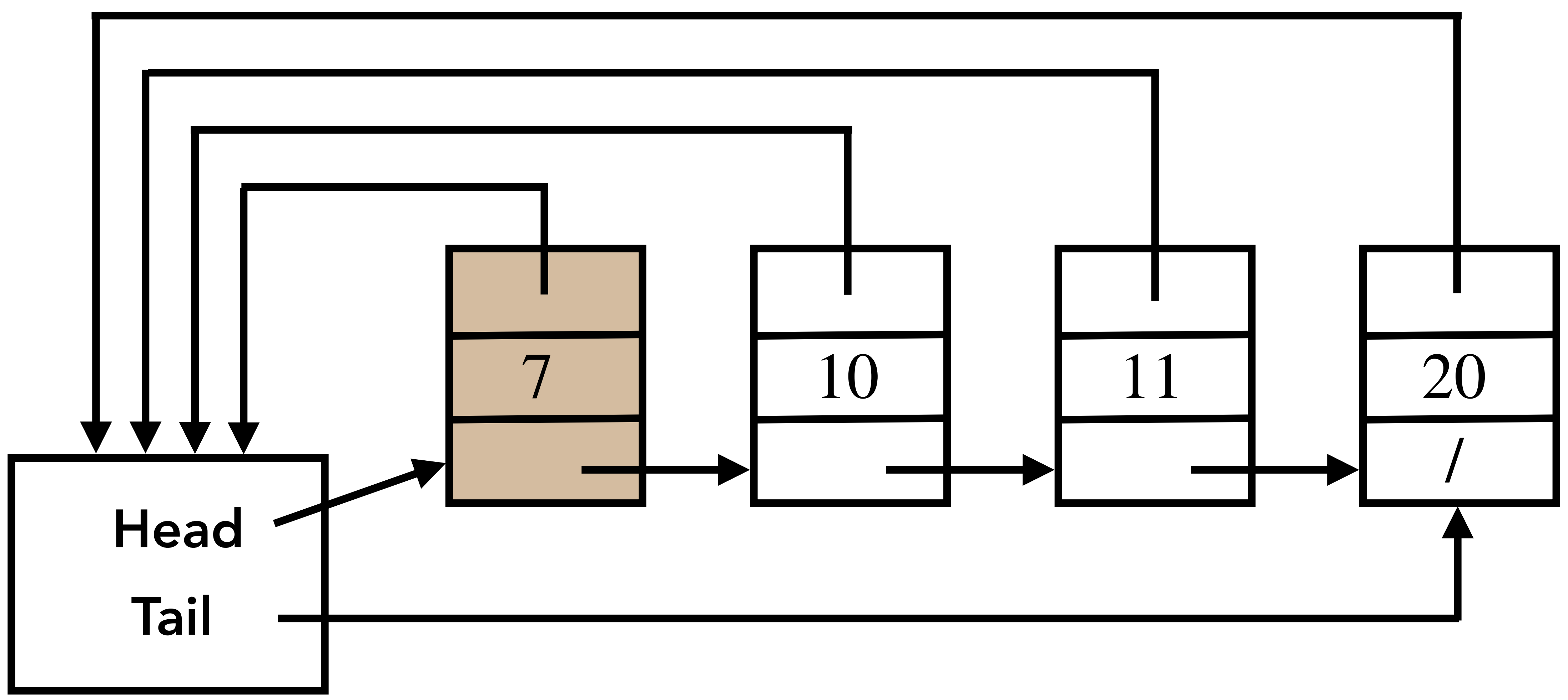

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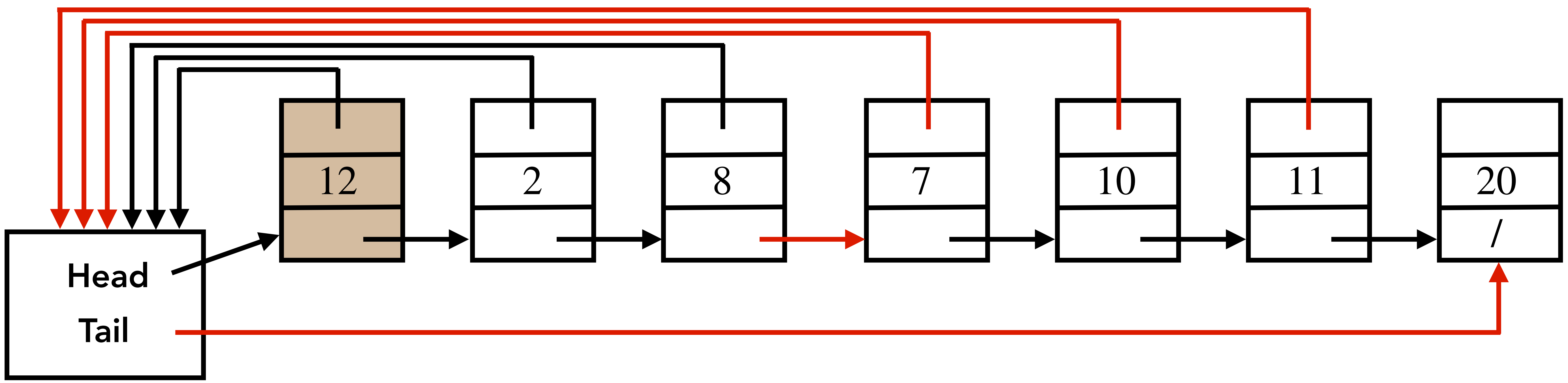

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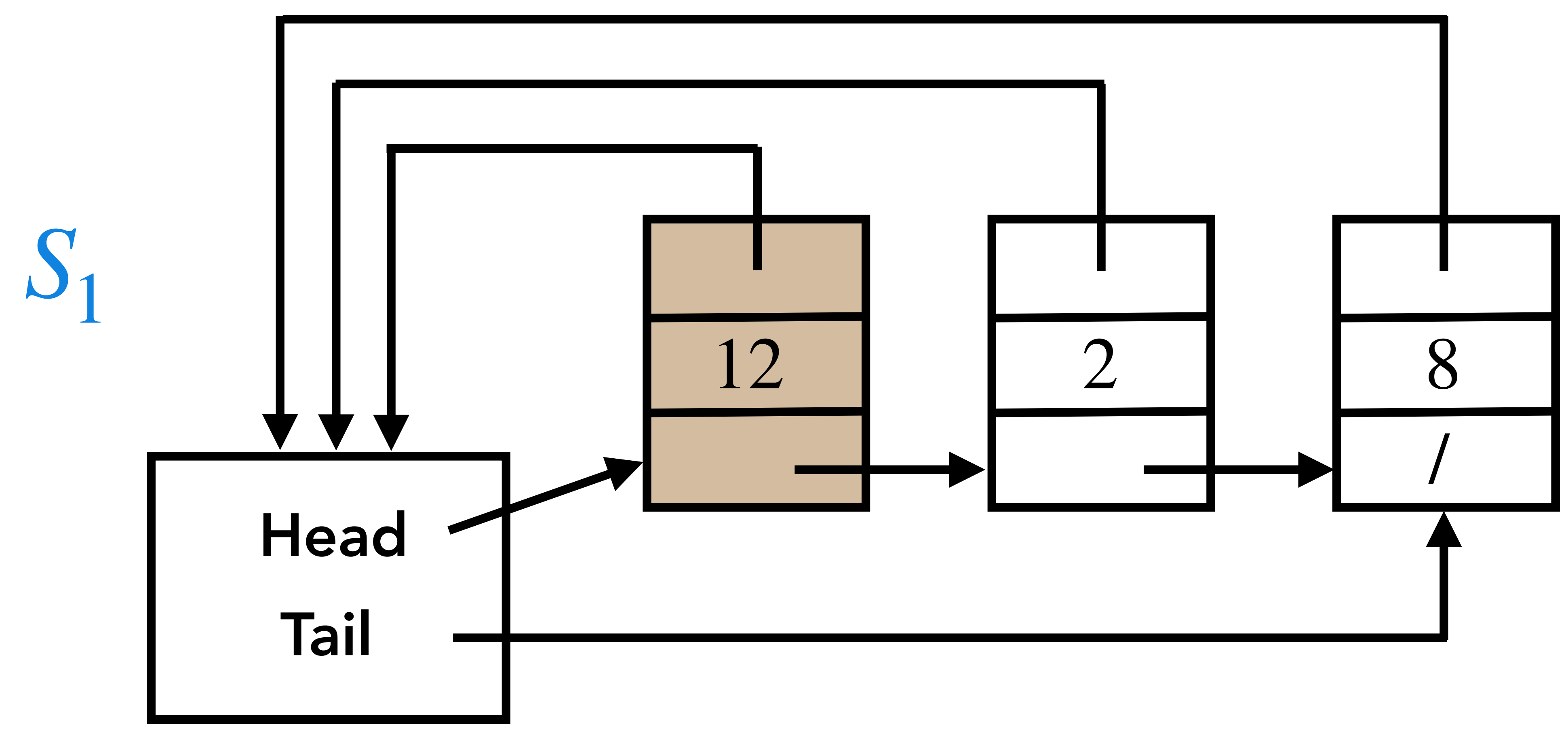

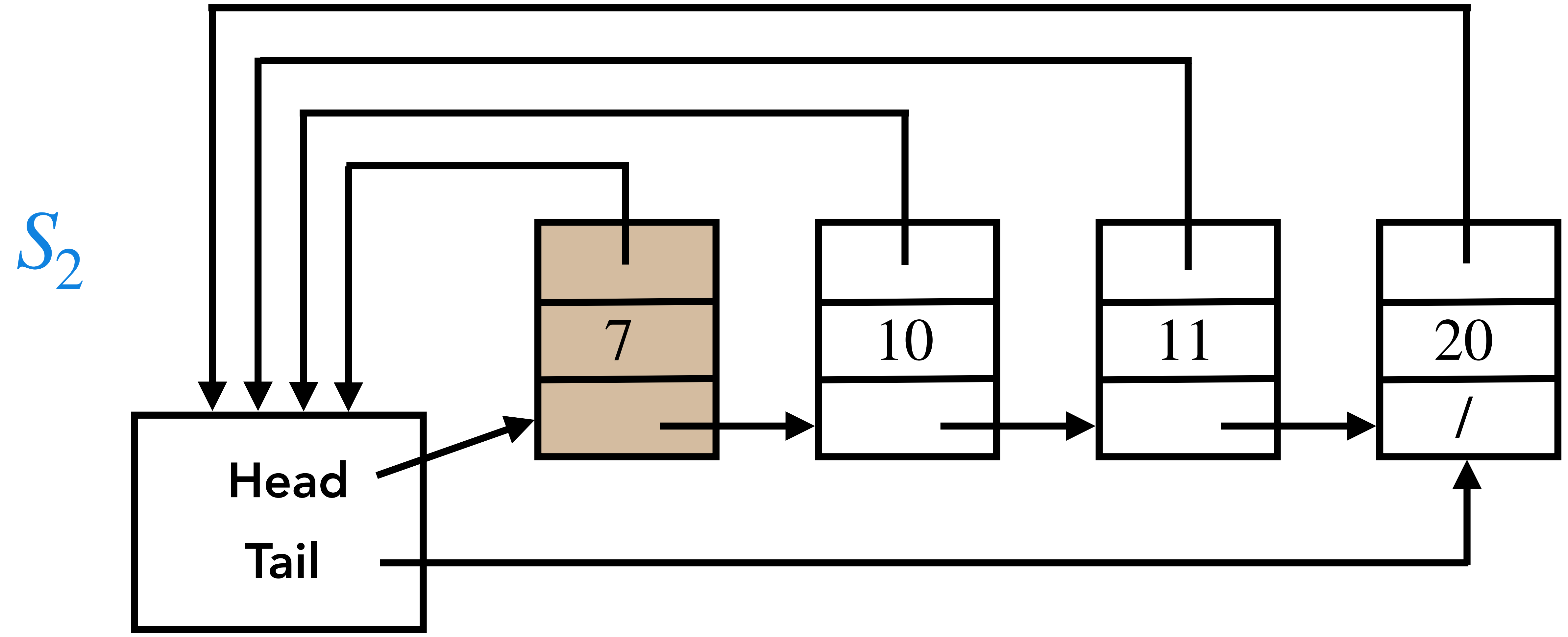

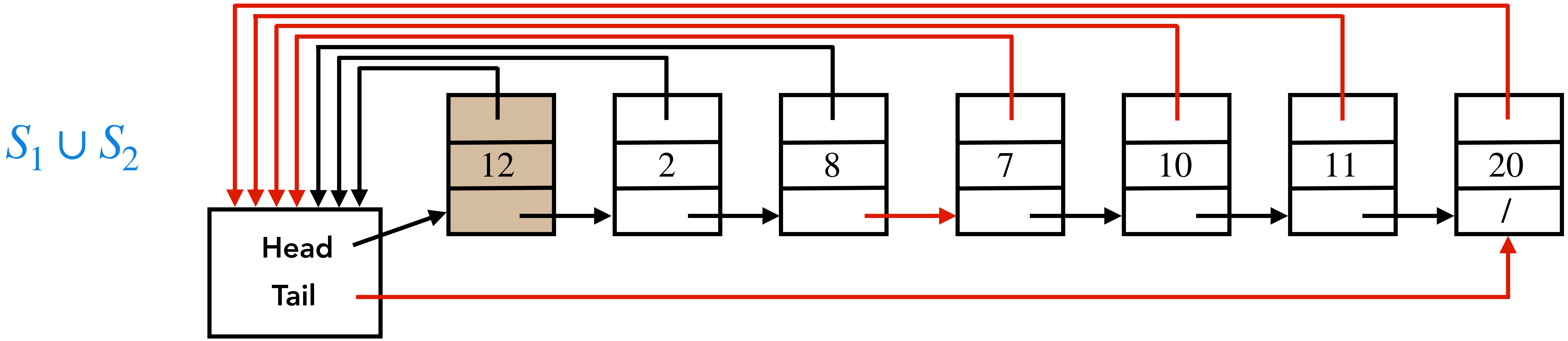

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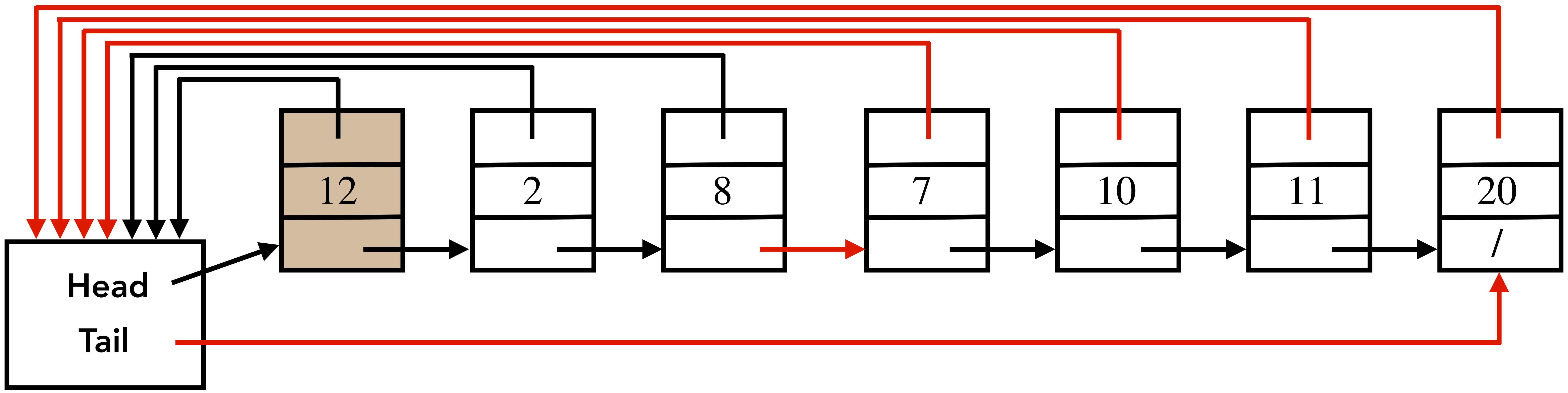

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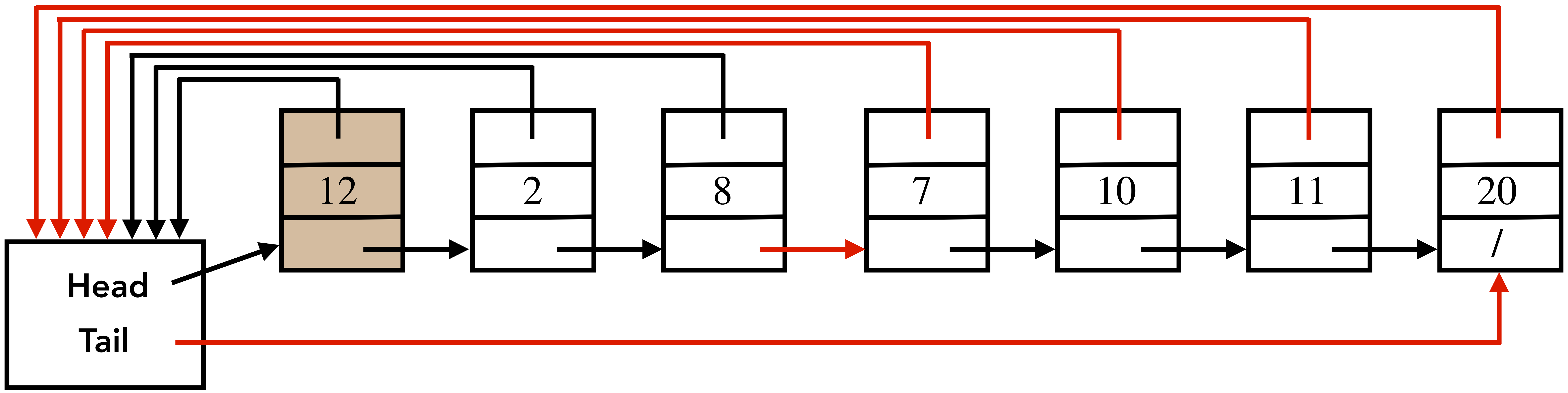

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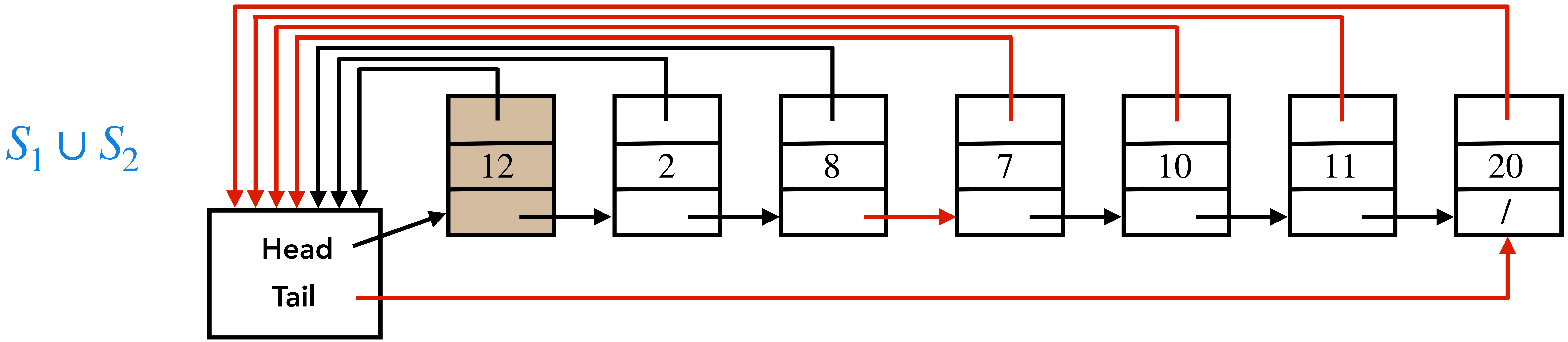

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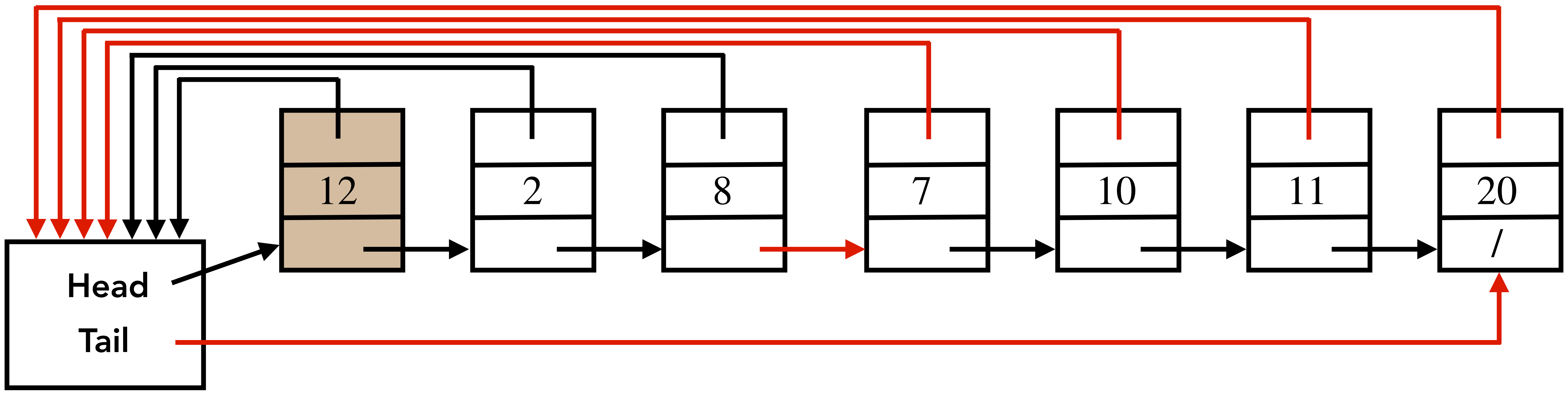

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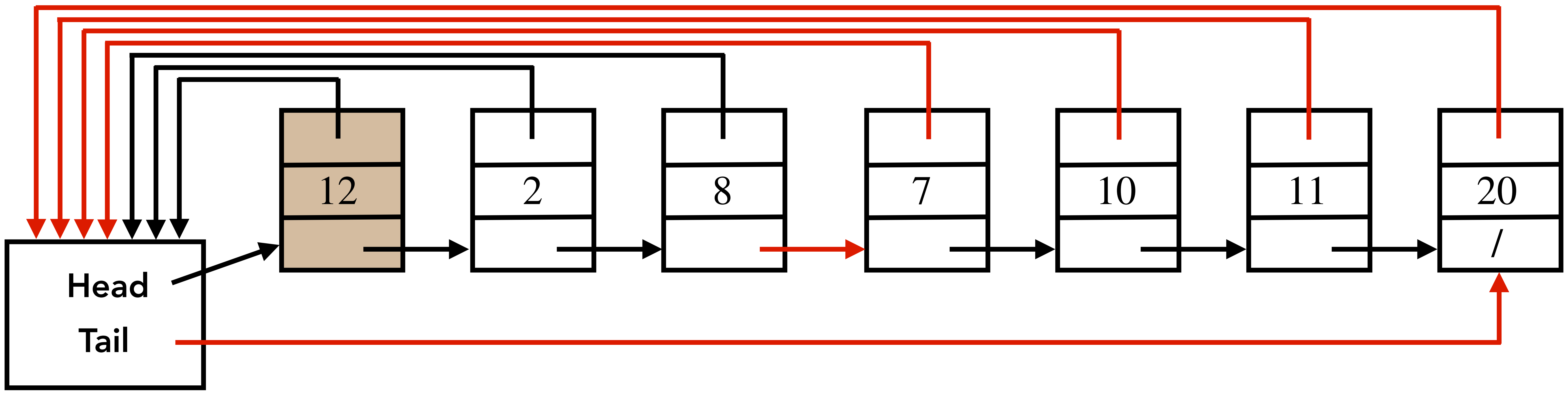

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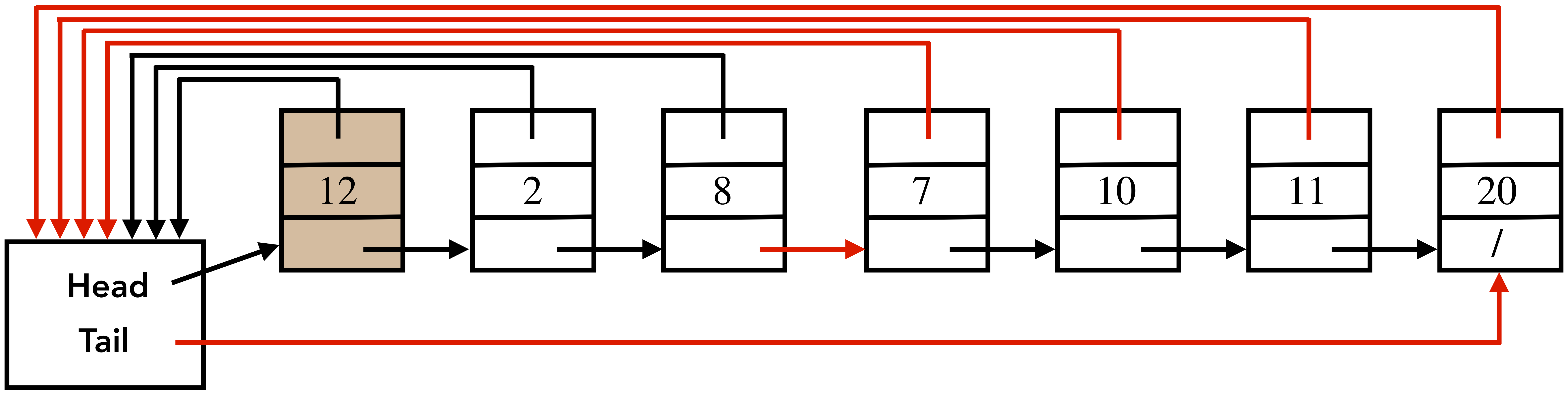

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Proof: DIY.

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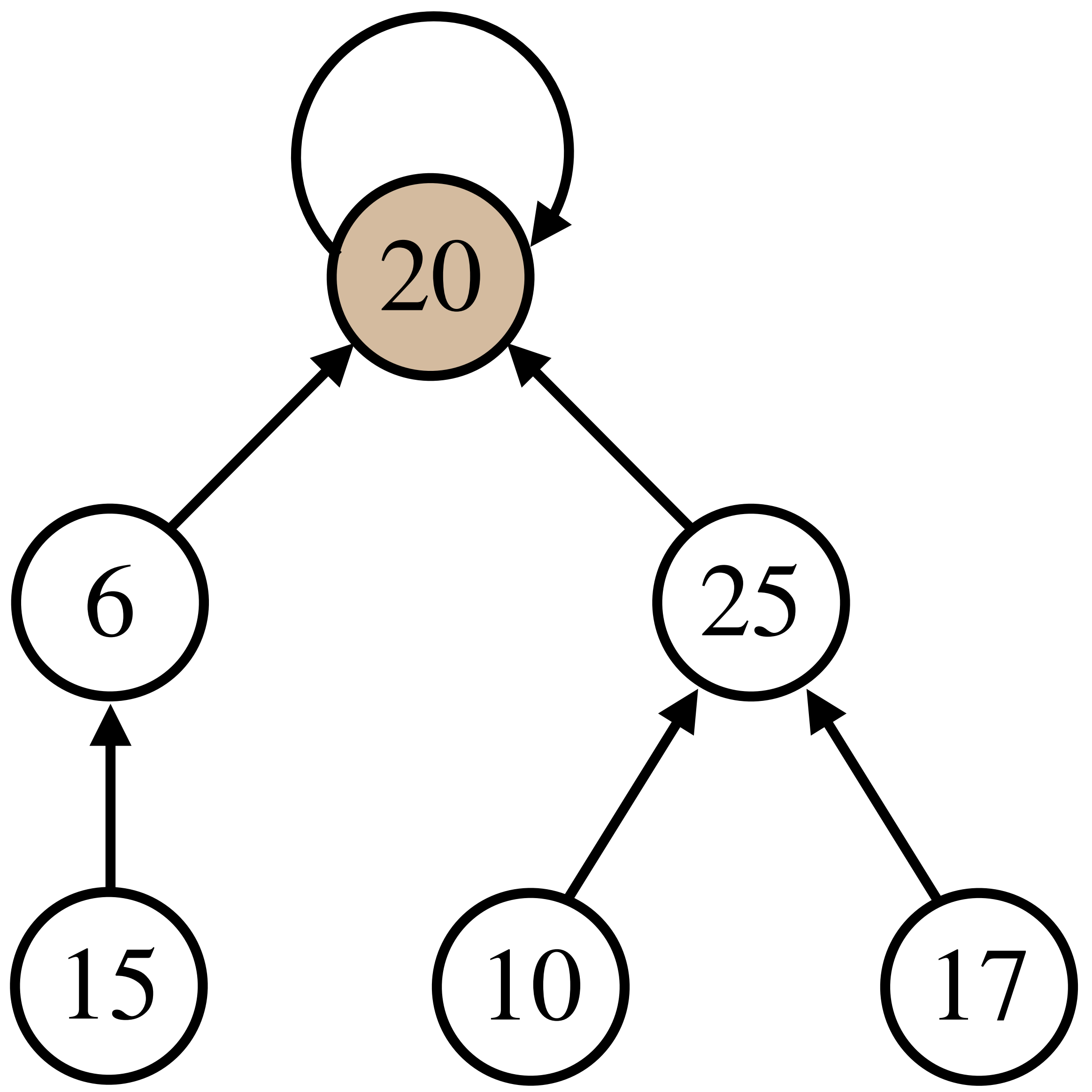

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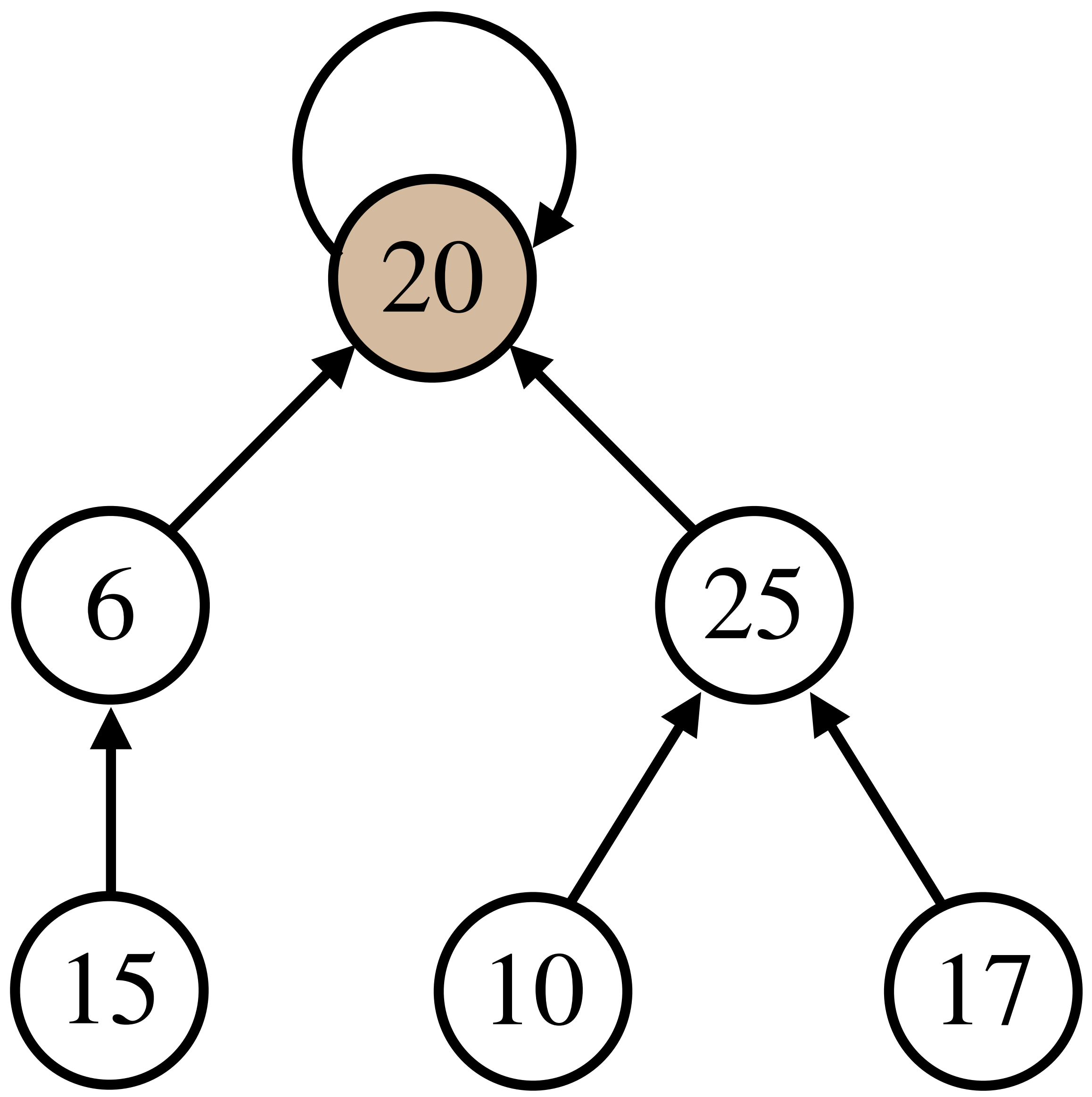

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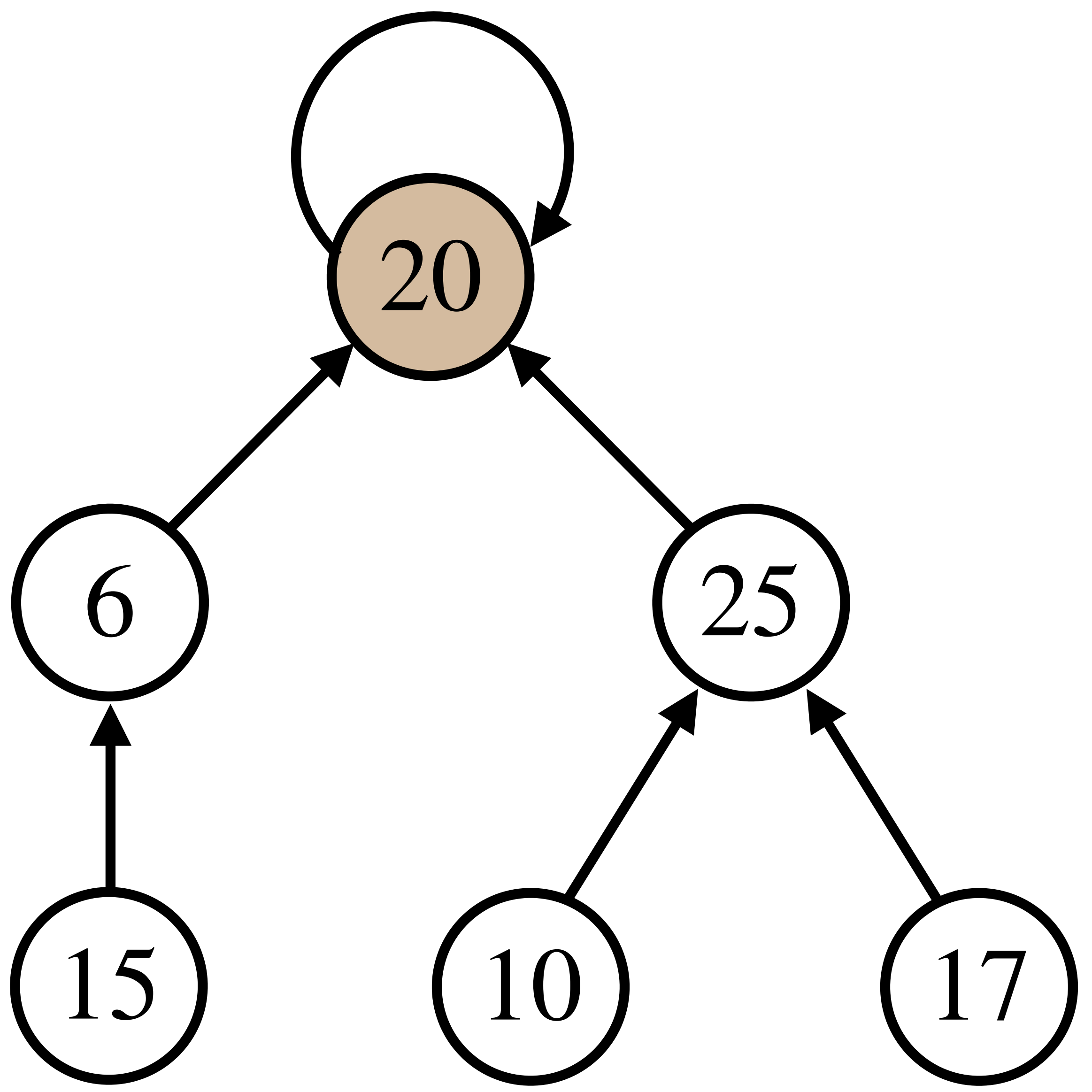

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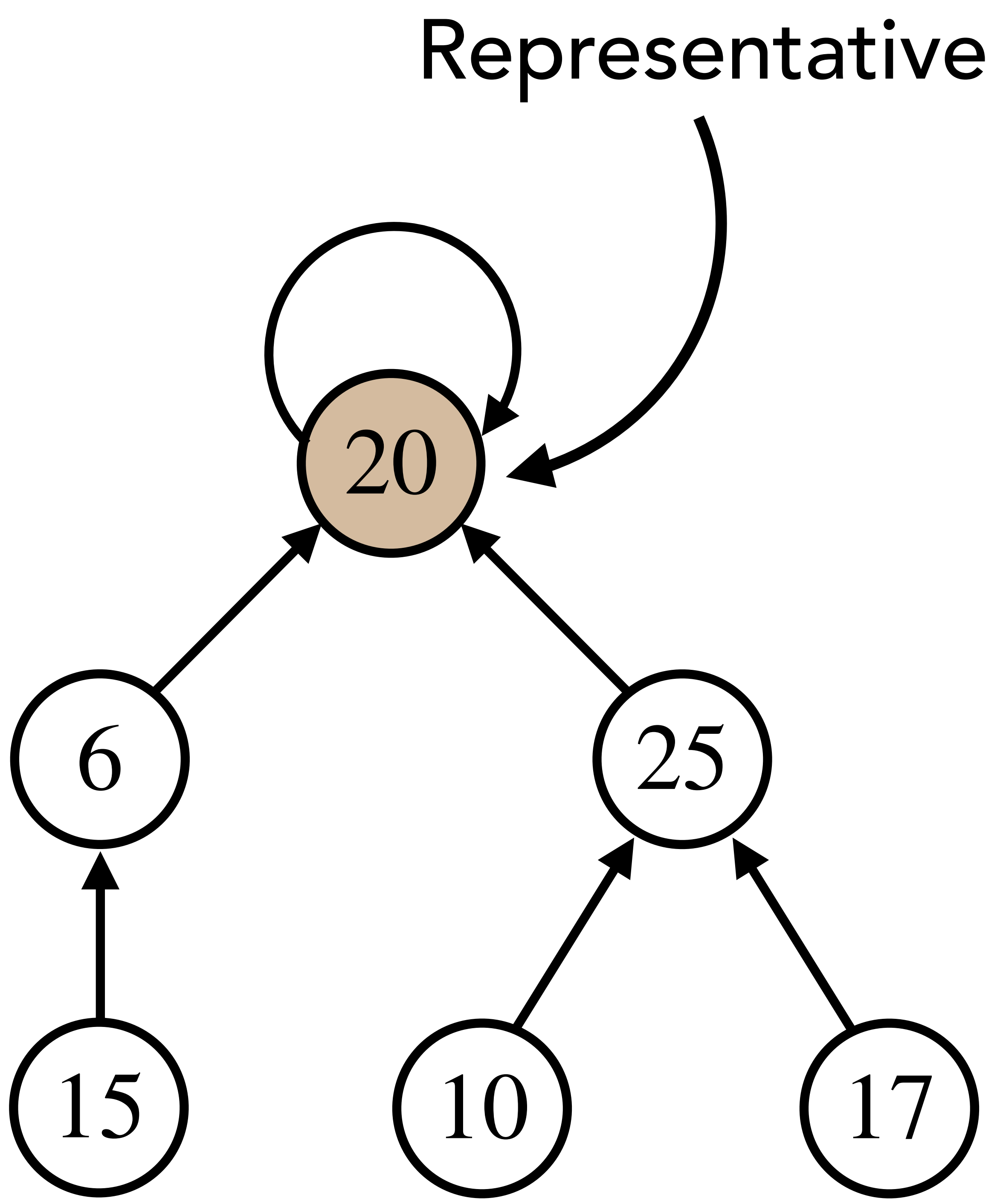

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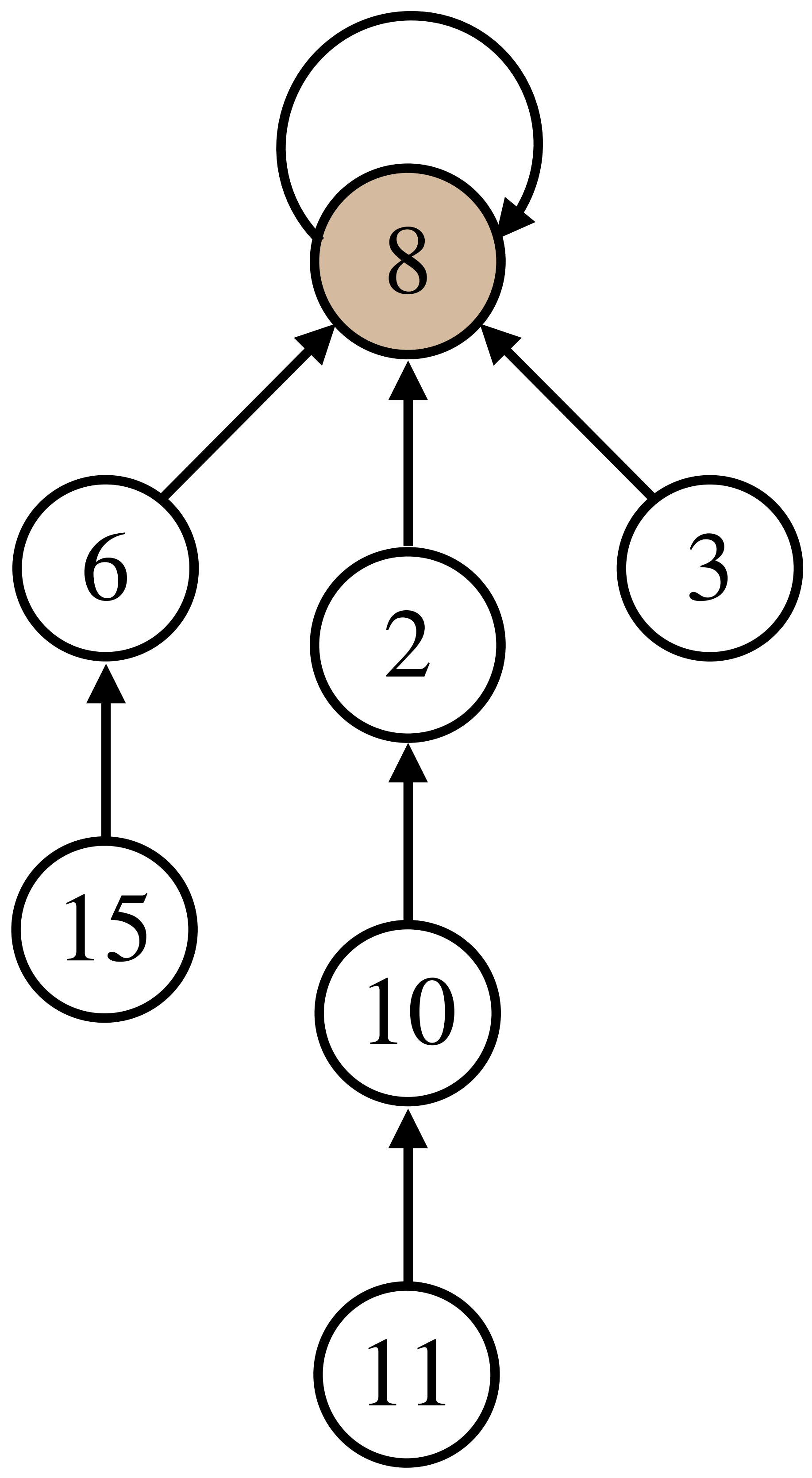

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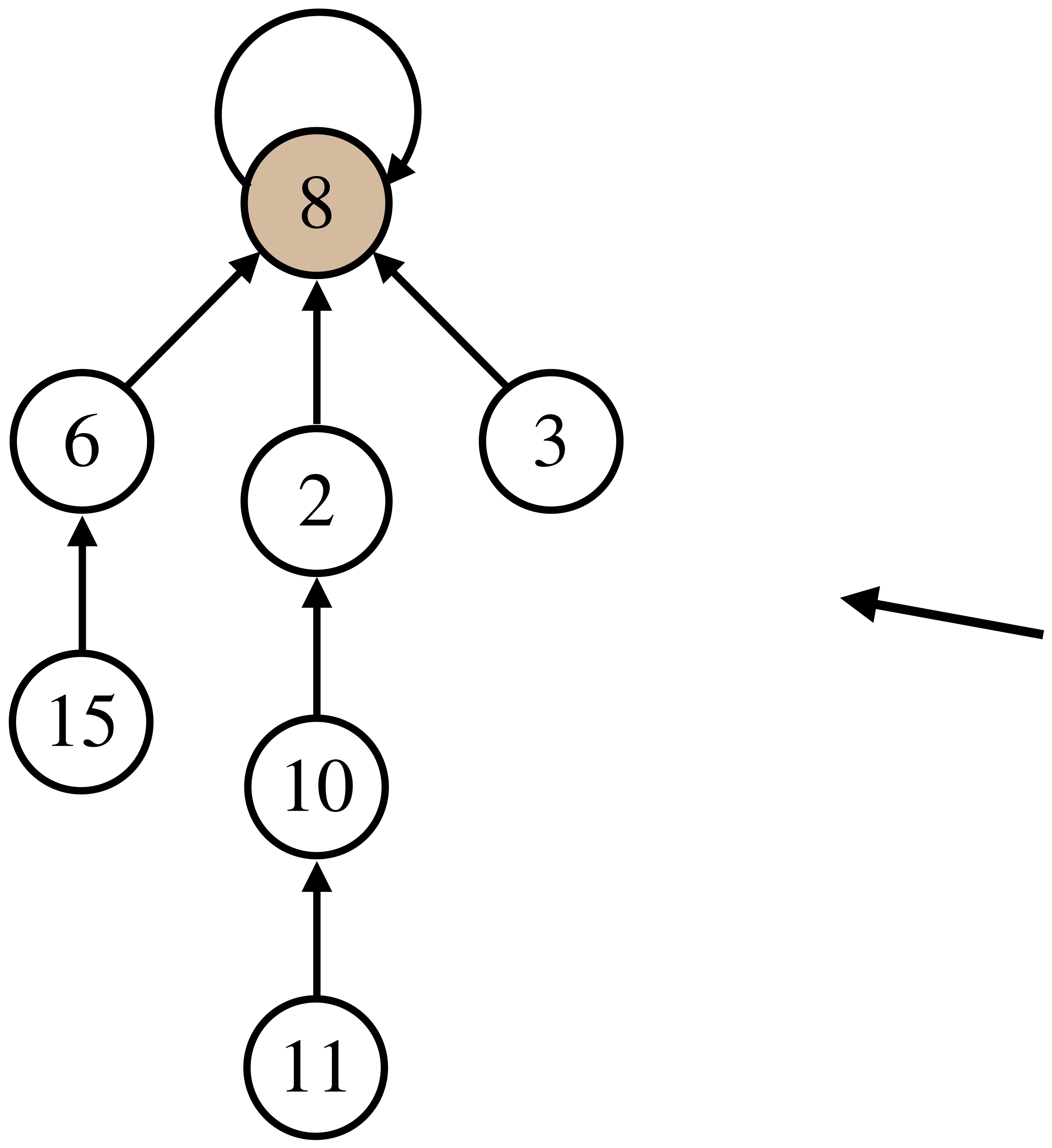

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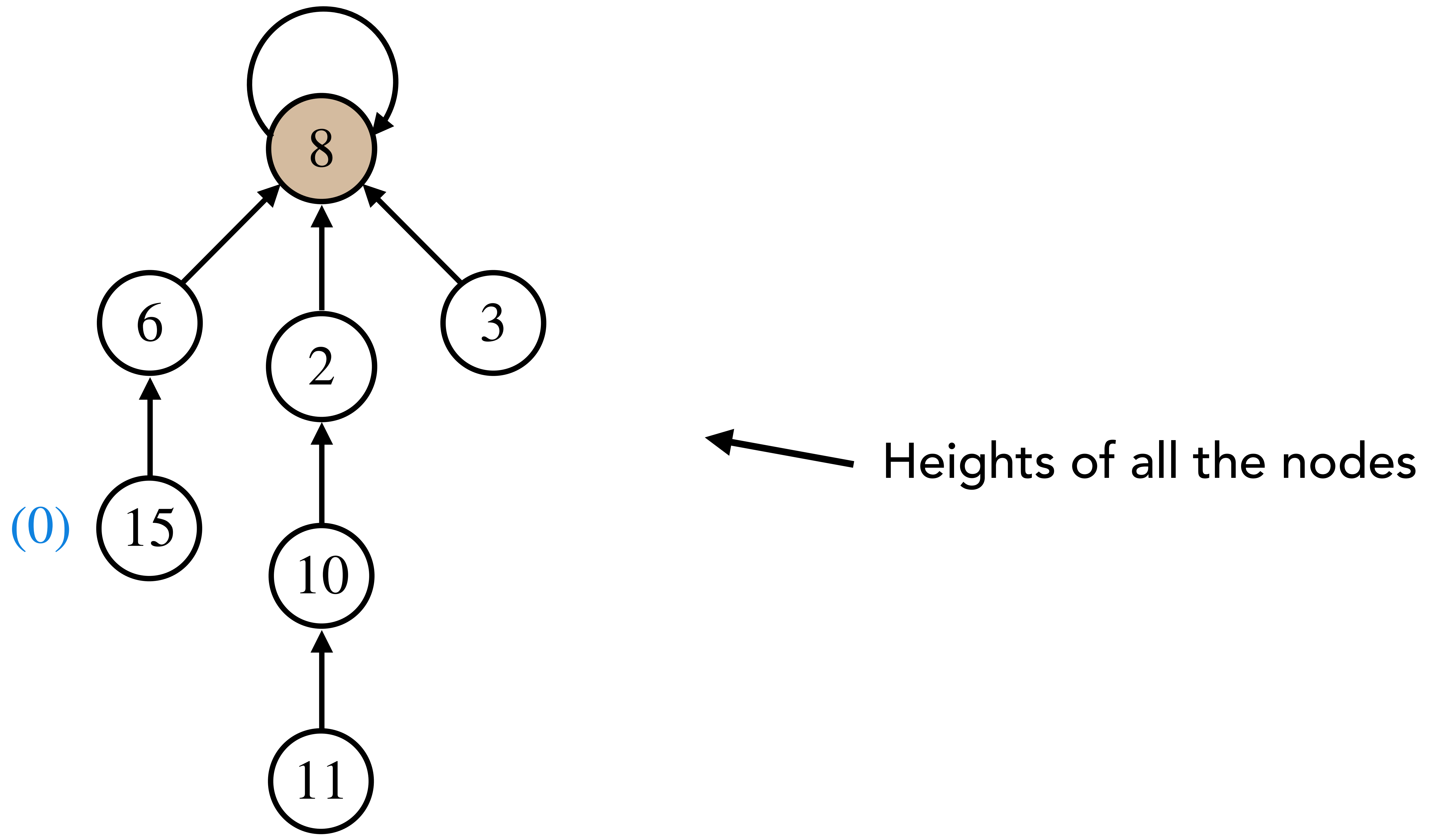

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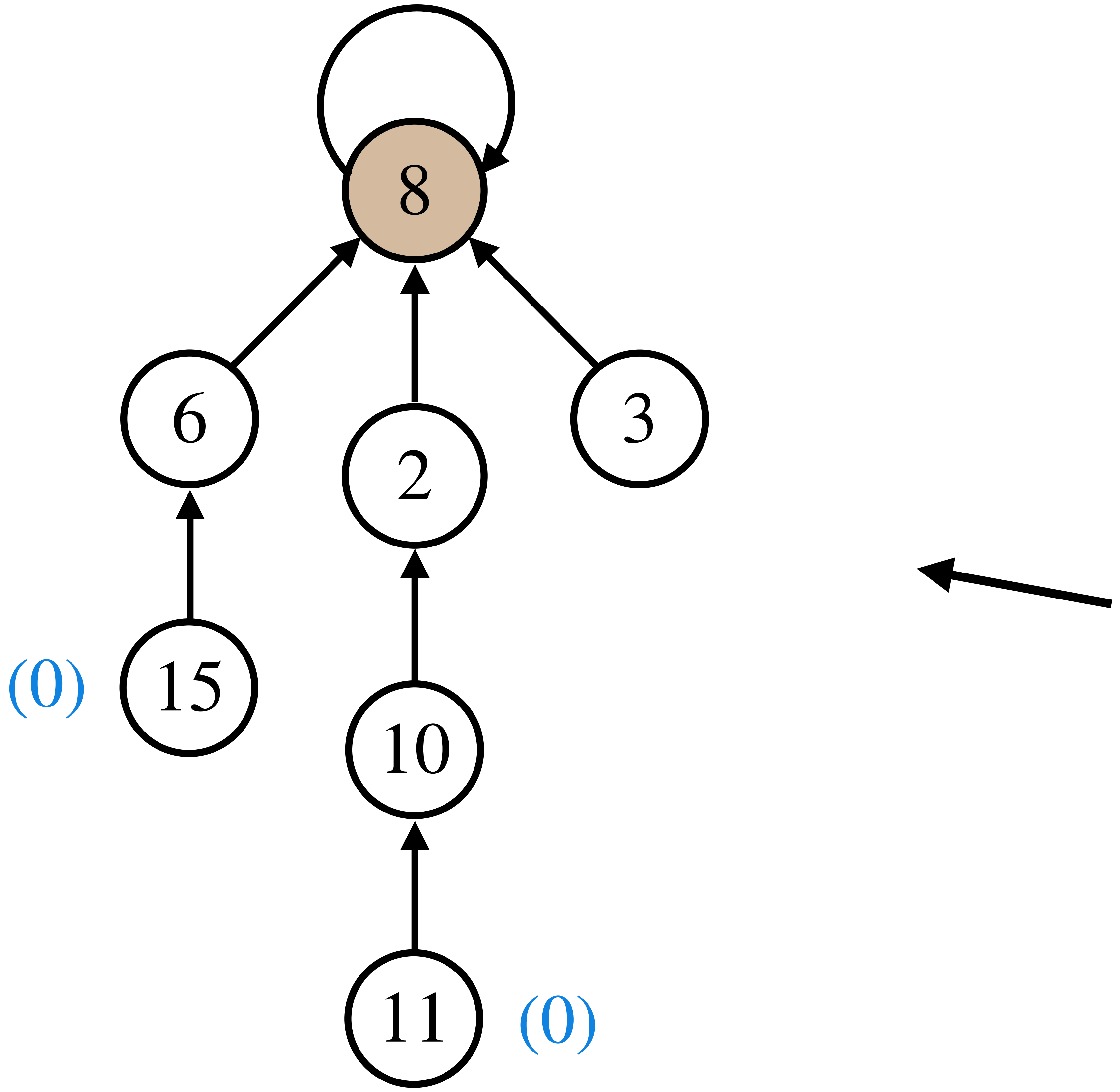

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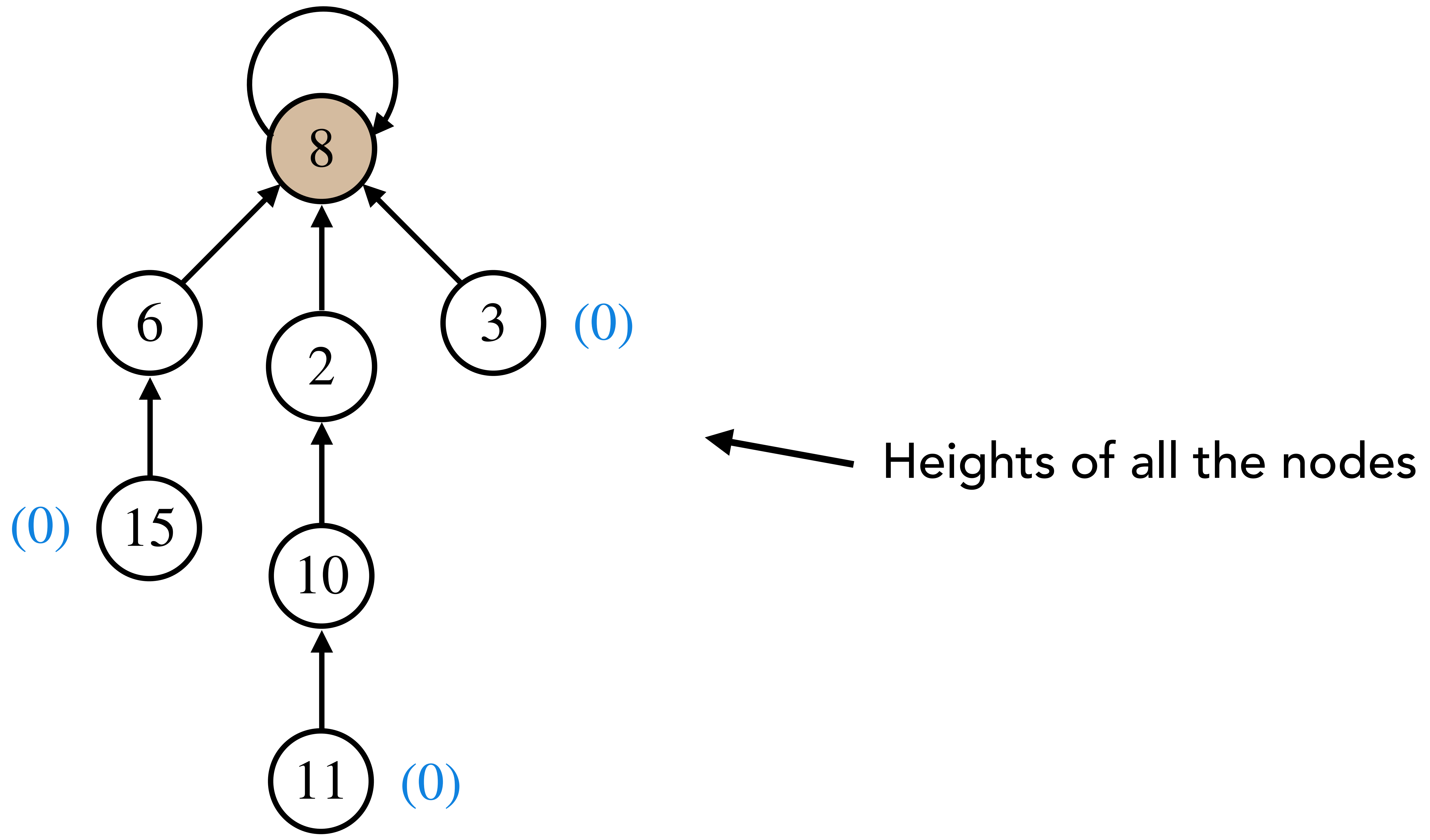
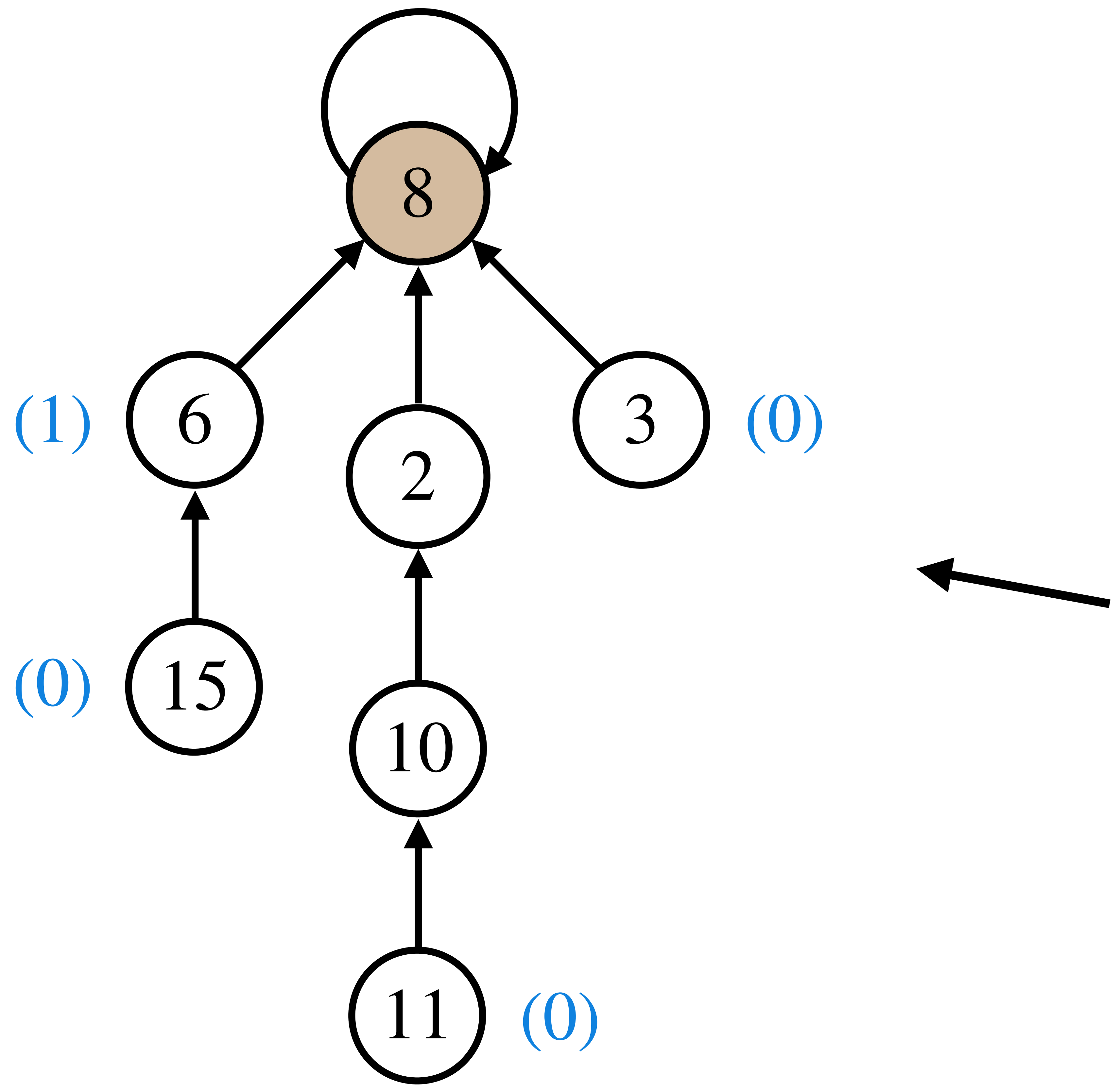


Height in Disjoint-Sets as Trees

Defn: Height of a node X in a disjoint set represented as a tree is the number of edges in the longest path from a descendant leaf to x.

Example:

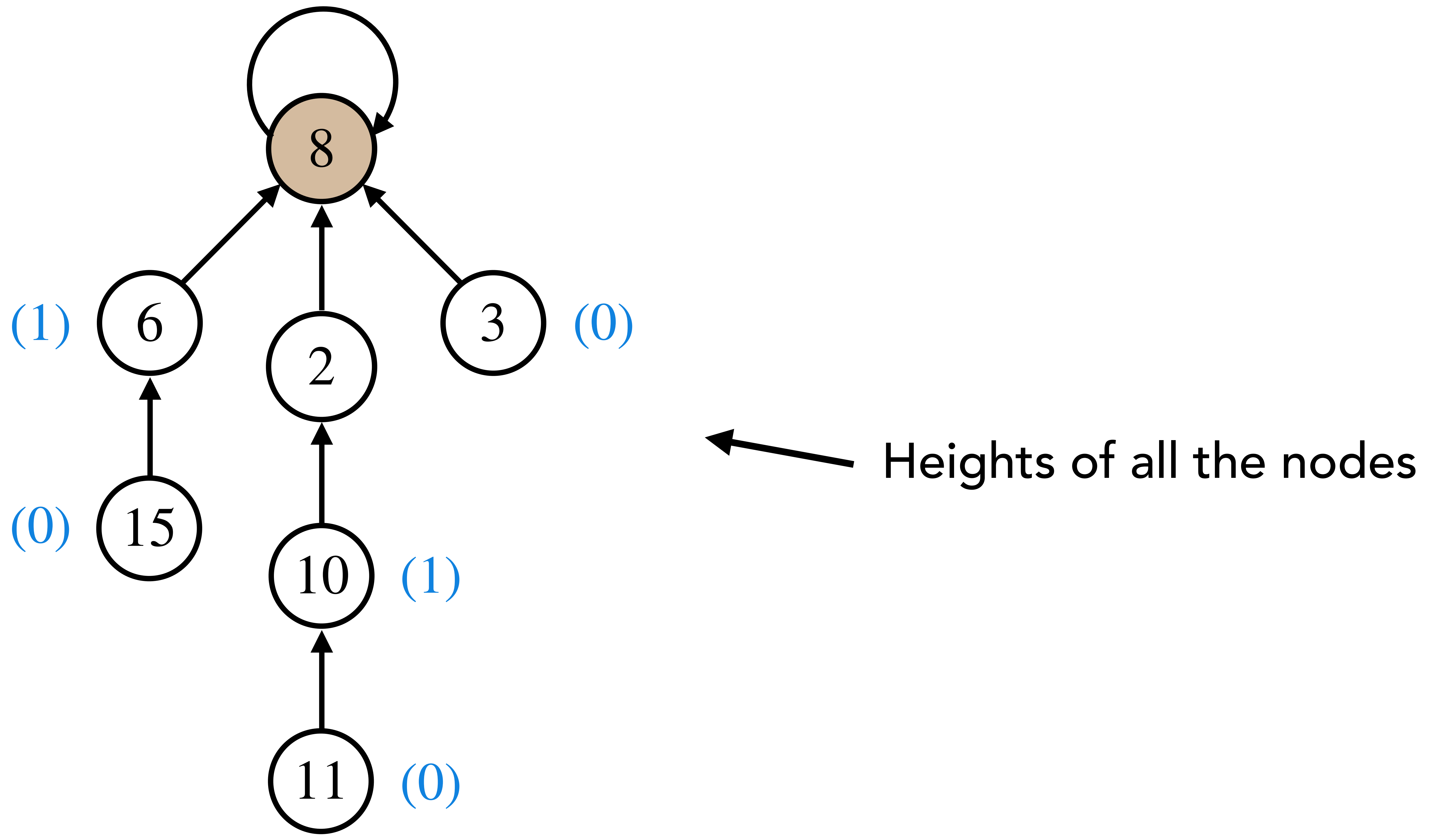


Heights of all the nodes

Height in Disjoint-Sets as Trees

Defn: Height of a node X in a disjoint set represented as a tree is the number of edges in the longest path from a descendant leaf to x.

Example:



Height in Disjoint-Sets as Trees

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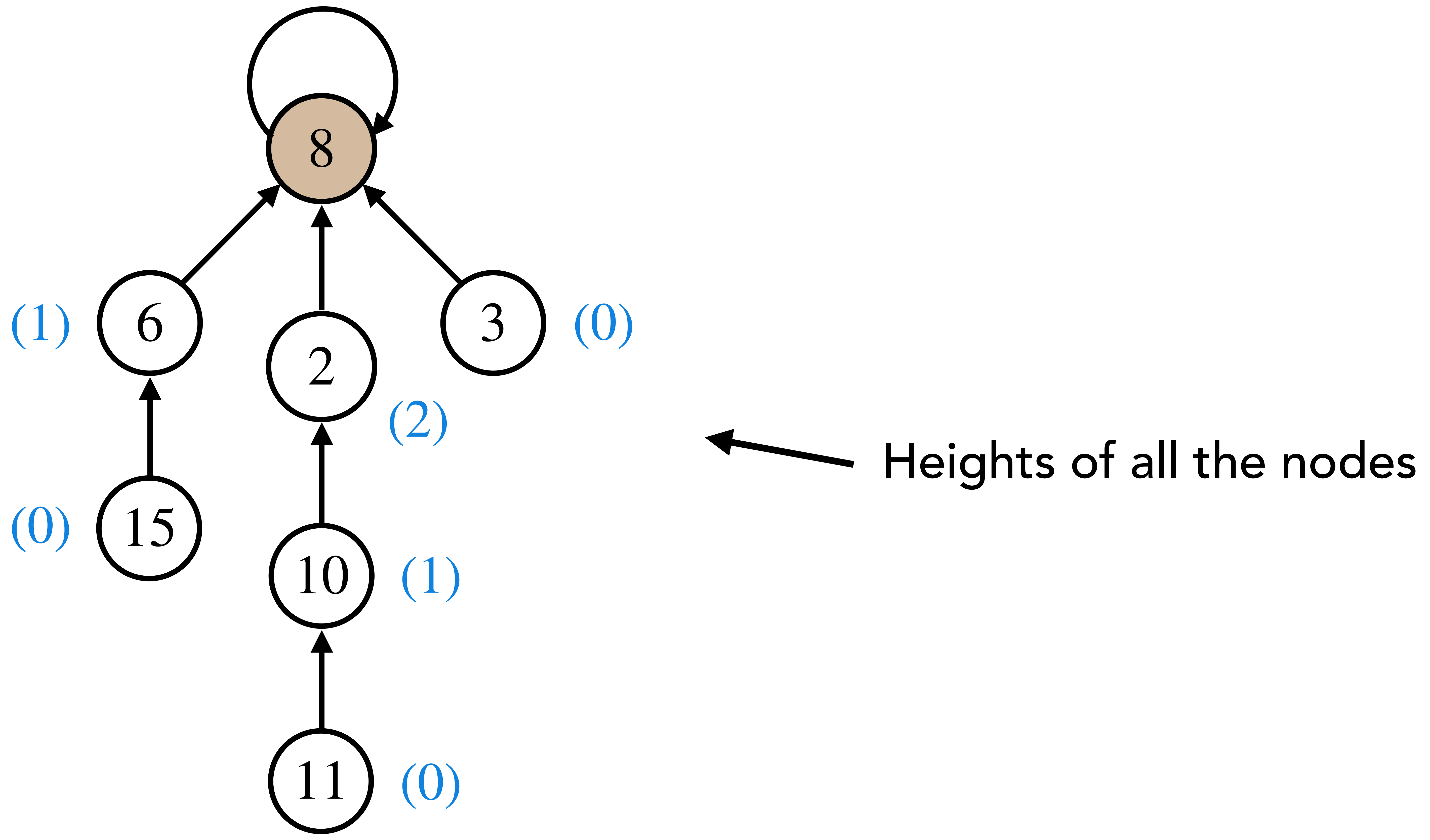
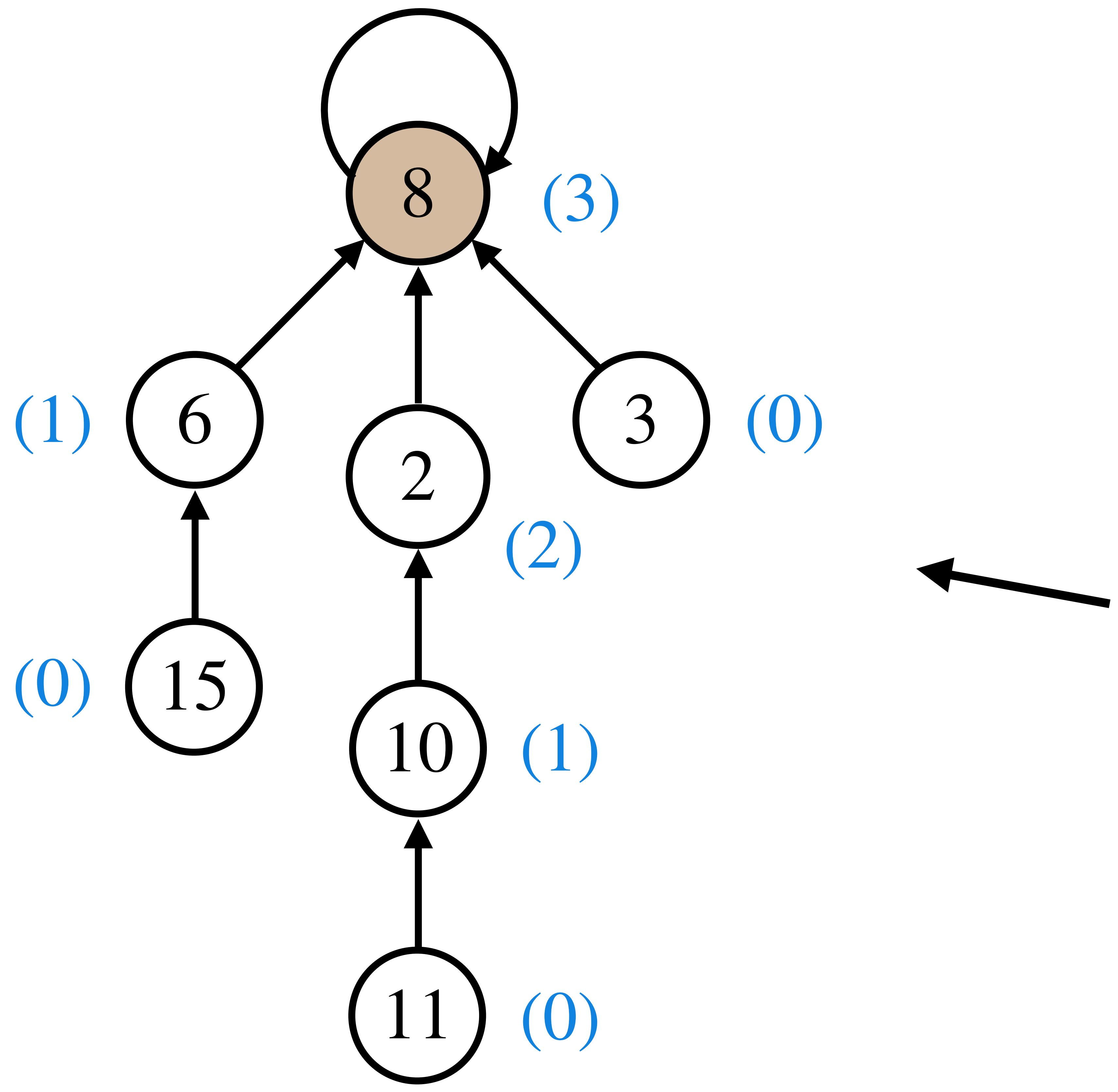
Example:



Height in Disjoint-Sets as Trees

Defn: Height of a node X in a disjoint set represented as a tree is the number of edges in the longest path from a descendant leaf to x.

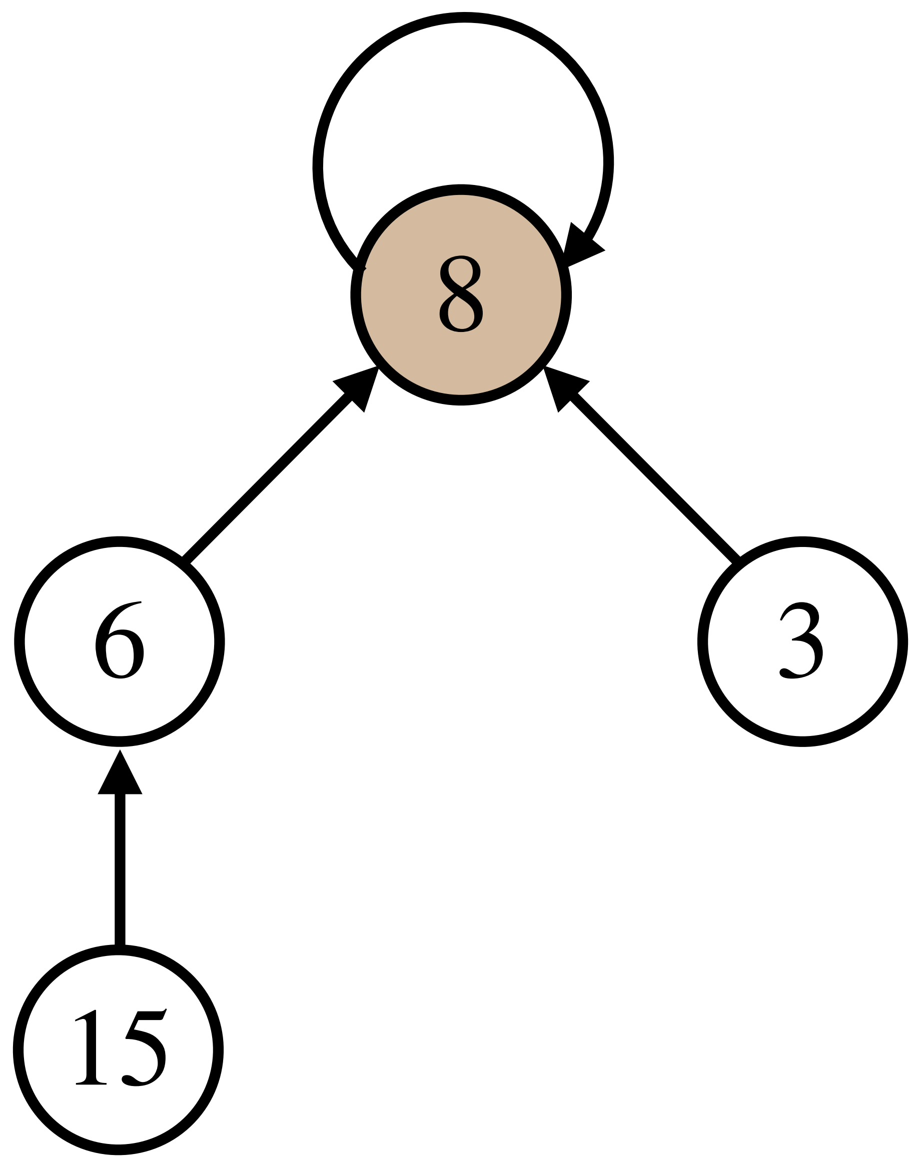
Example:



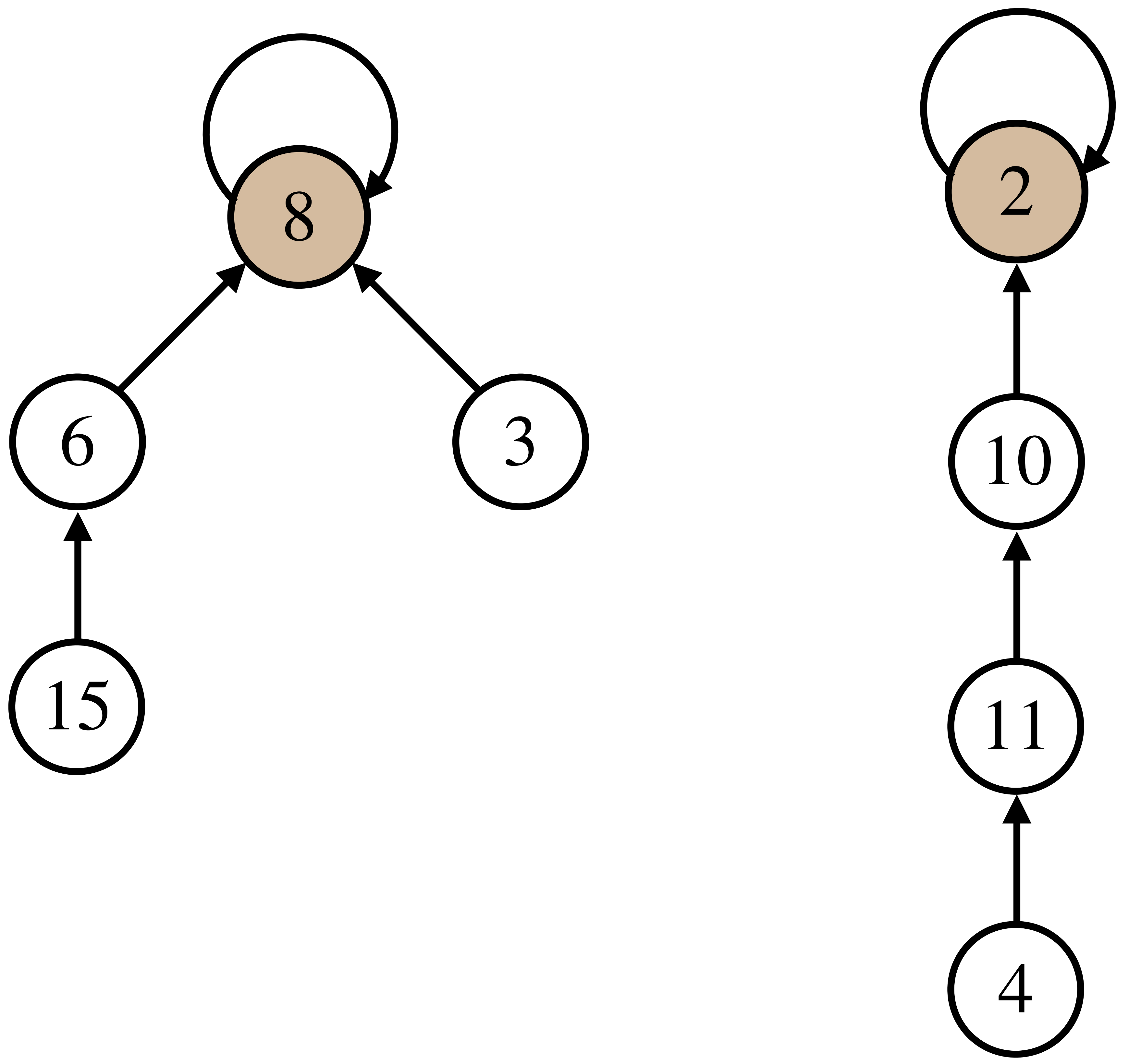
Heights of all the nodes

Union on Disjoint-Sets as Trees

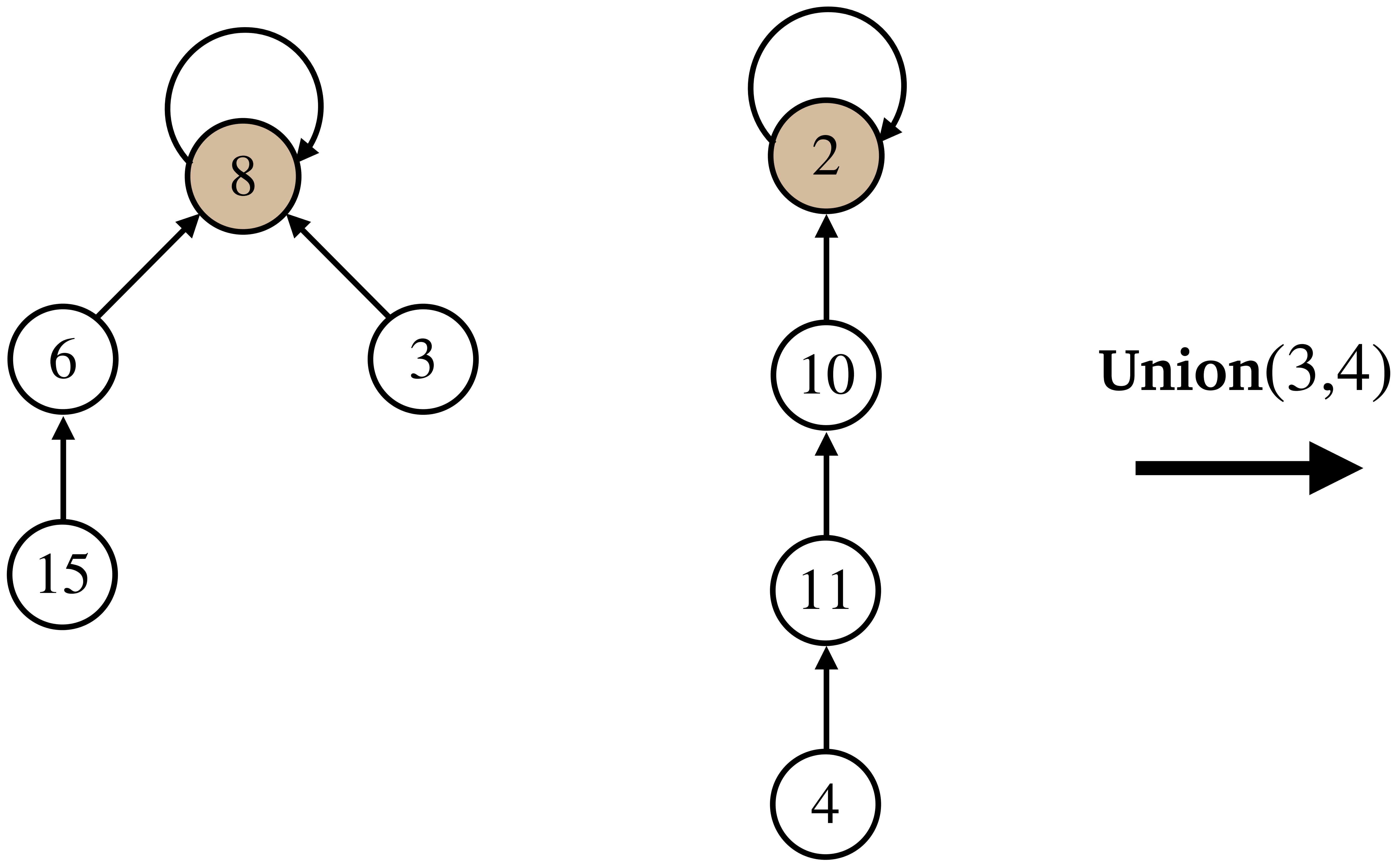
Union on Disjoint-Sets as Trees



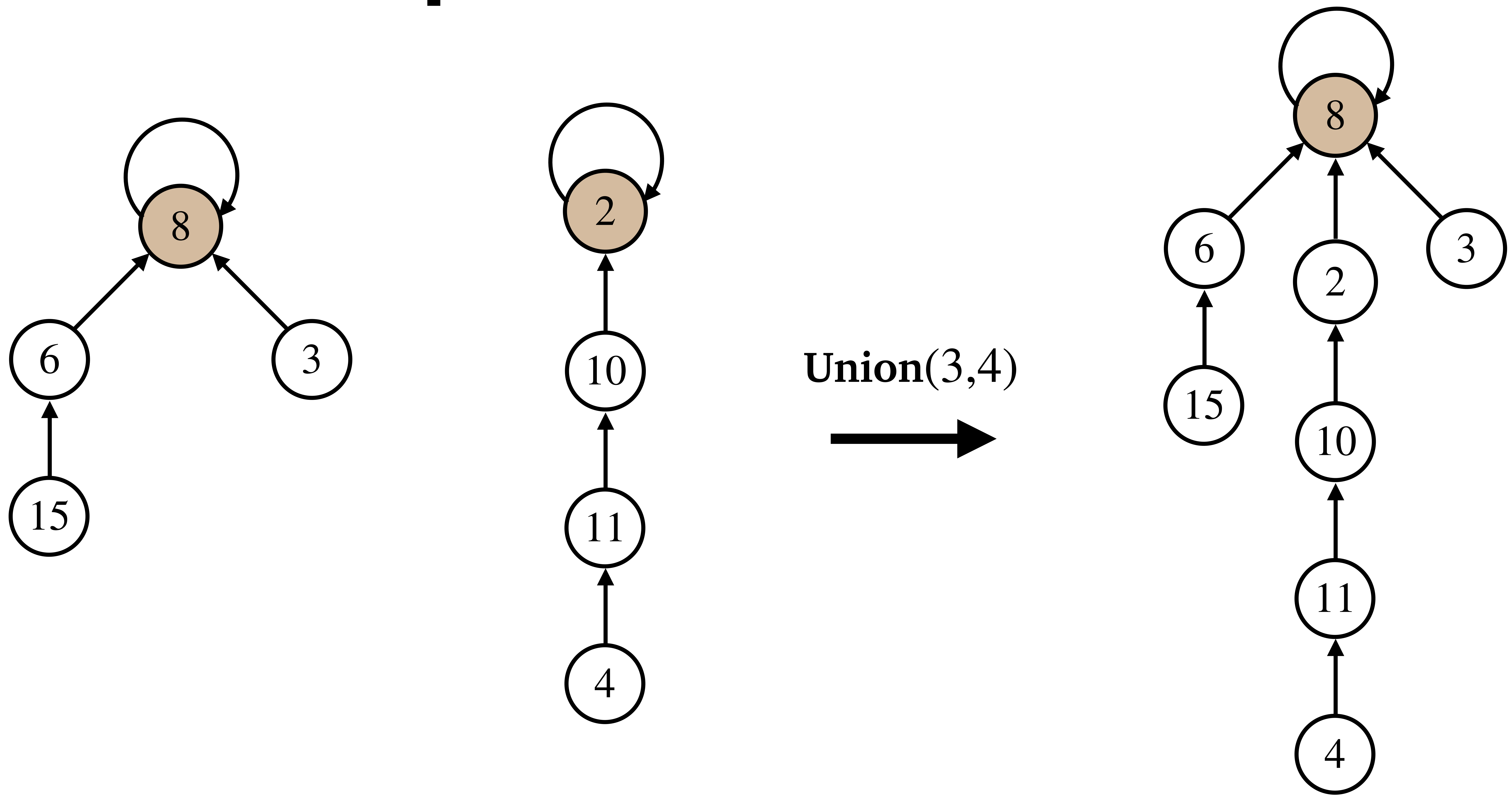
Union on Disjoint-Sets as Trees



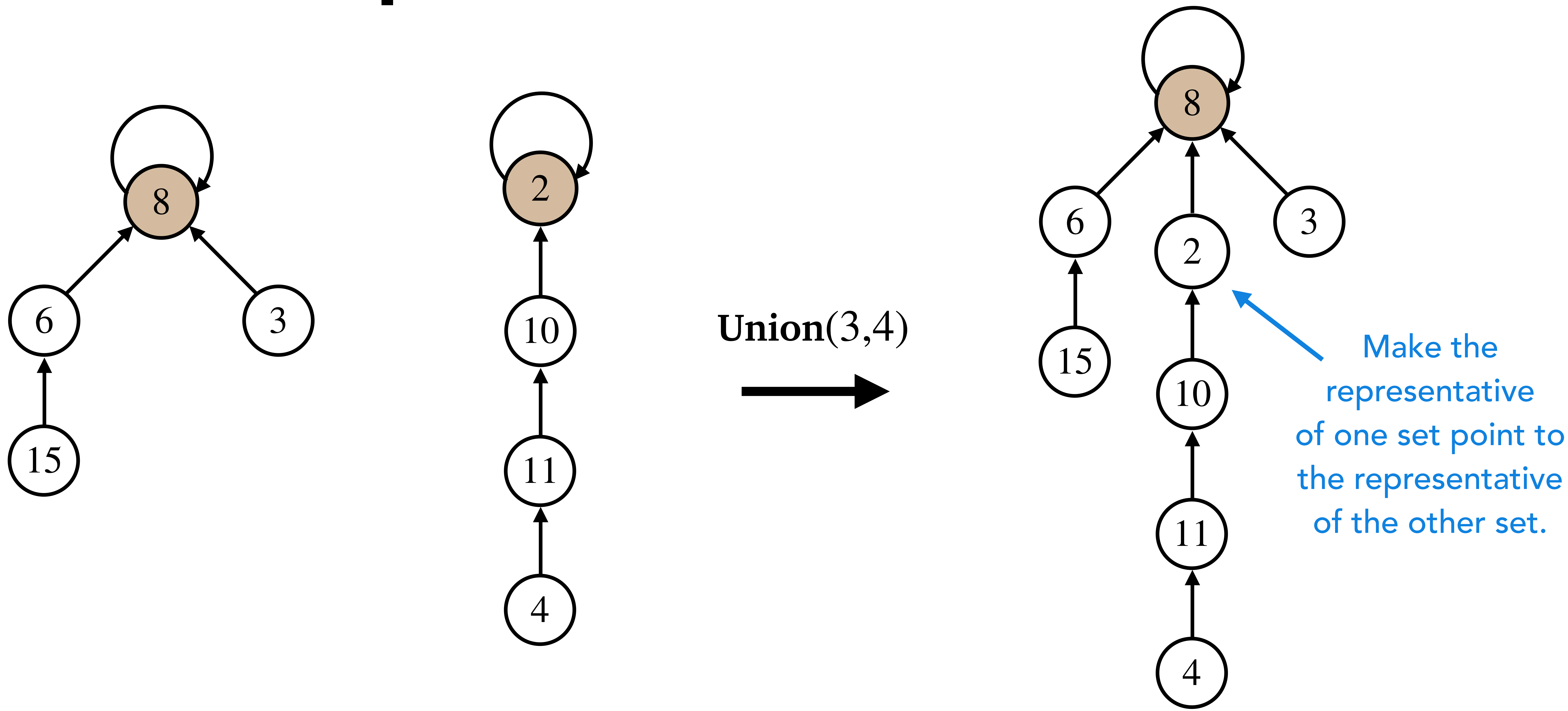
Union on Disjoint-Sets as Trees



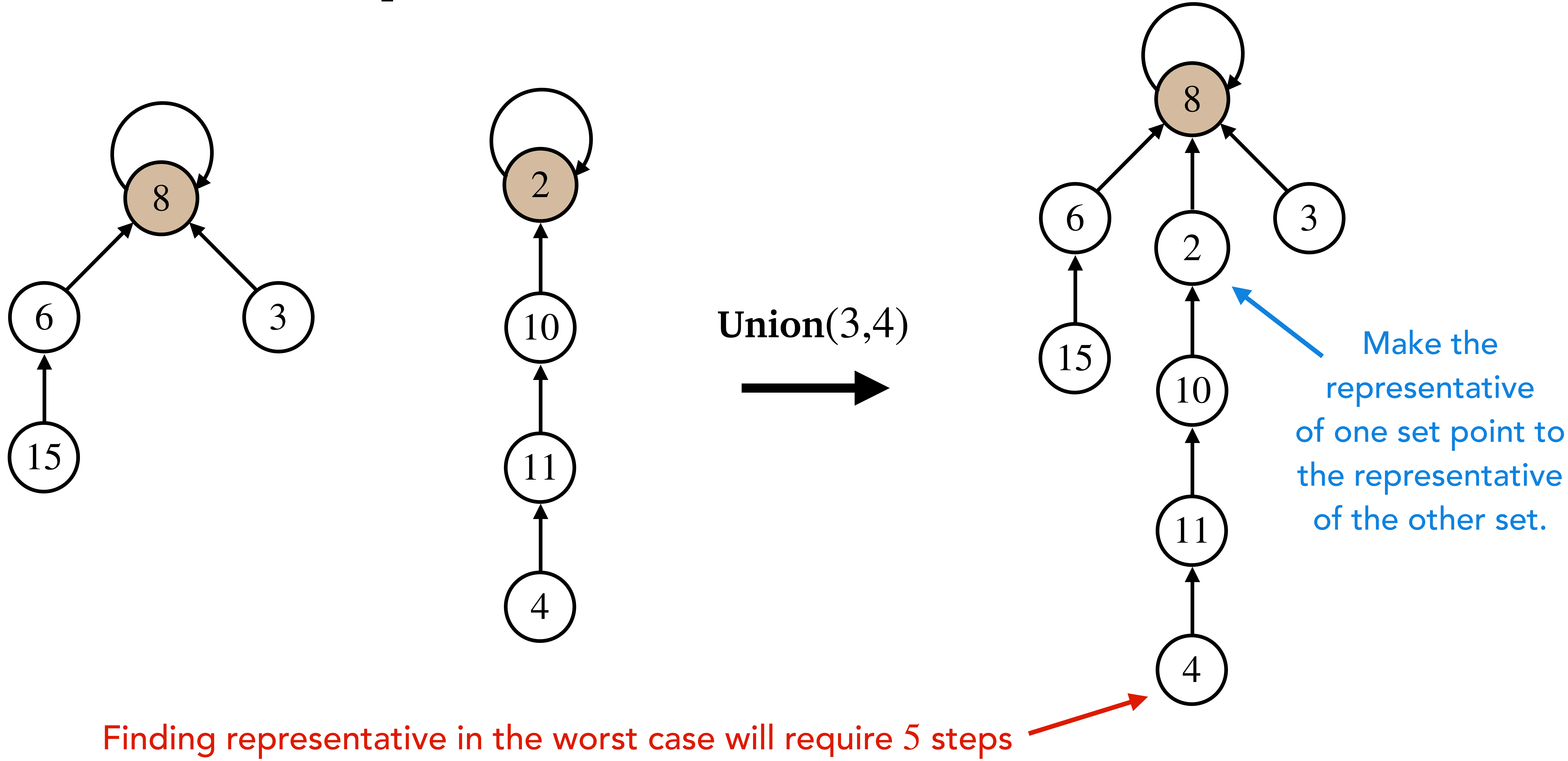
Union on Disjoint-Sets as Trees



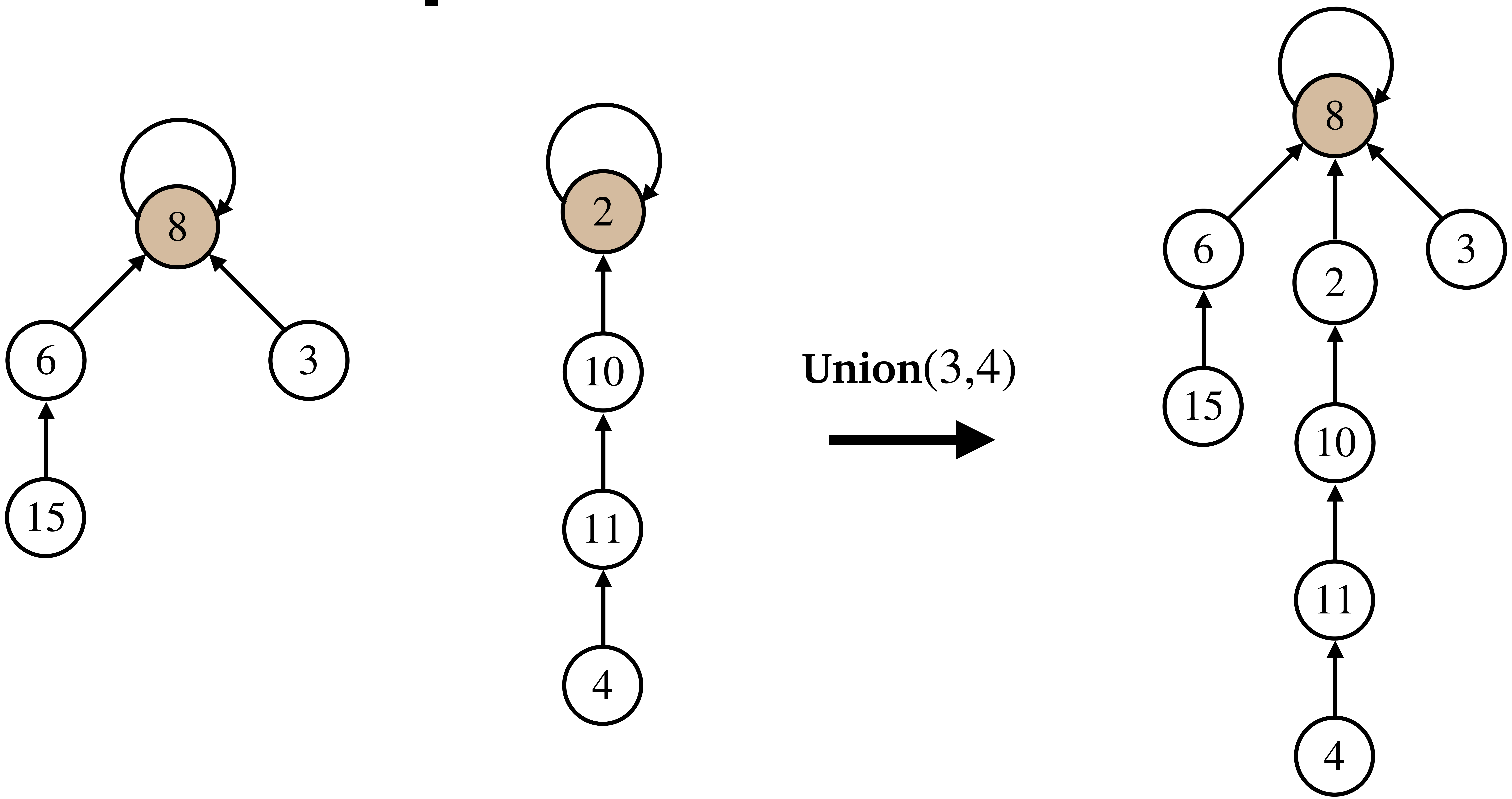
Union on Disjoint-Sets as Trees



Union on Disjoint-Sets as Trees

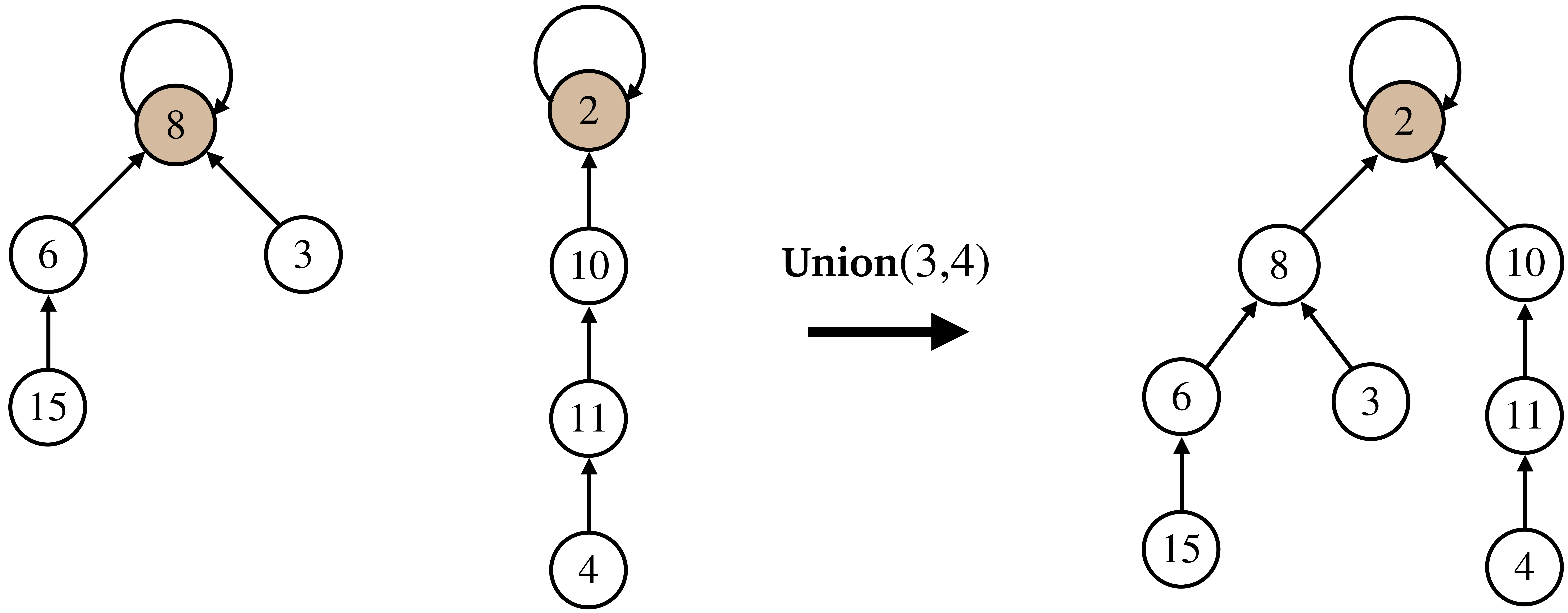


Union on Disjoint-Sets as Trees



Shouldn’t representative with smaller height point to representative with larger height?

Union on Disjoint-Sets as Trees



Union on Disjoint-Sets as Trees